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Vol. 228 **SEPTEMBER, 1939** **No. 3**

CONTENTS

Science and Human Affairs ..... W. F. G. SWANN 263

Progress in Metallurgy: The Science of Alloys ..... A. B. KINZEL 293

Beat Theory of Non-linear Circuits ..... E. G. KELLER 319

Graphical Analysis of Transient Phenomena in Electric Circuits
KALMAN J. DEJUHASZ 339

Notes from the National Bureau of Standards 375

The Franklin Institute Notes 385

Book Reviews 408

Publications Received 409

Current Topics 292, 338, 374, 384

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SCIENCE AND HUMAN AFFAIRS.*

BY

W. F. G. SWANN,

Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pennsylvania.

Introduction. A thousand million years ago, our sun—a star, like billions of its kind—stood lonely in the universe. There was no earth or other planet to pay it homage, or seek sustenance from it. It stood in the emptiness of space awaiting its destiny and separated by three million million miles from its nearest neighbor. And then some other star or wandering comet happened to come within the scope of its influence. The visit was but for an instant, as astronomy counts time; but in that instant the forces of gravitation raised great tides upon the sun, shattered the substance of its surface, tearing away those fragments which now we know as the planets and satellites of the solar system. That ancient visitor, whose advent meant so much, departed, but the results of his visit remained, and in a form destined to evolve a history which could find no place in the fires of the sun itself.

And then these new-born children of the sun spent many eons in adolescence, in which they could but reflect the properties of their parent, until they had cooled to such tem-

* A few phrases in this address have been quoted from the author's former writings upon related matters, and in particular from his "Architecture of the Universe" (Macmillan Company, New York, 1936).

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

peratures as made new experiences possible. Eventually, we know not how, life appeared; and in about the last one thousandth part of the period between the earth's birth and today, in the last million years or so, that life developed to the form of man.

Man, at least the man of our solar system, is truly but an infant in the universe. The light by which our astronomers see the most distant stars today left those stars a hundred million years ago, and traveling with the speed of 186,000 miles a second, had traveled ninety-nine hundredths of its journey to us by the time that the first man on earth was born. Truly, the time of man's existence on the earth is far less than the time of an echo of Nature's voice in the universe.

Man's early history lies, for the most part, imbedded in the rocks of antiquity, and only concerning the most recent one per cent. or less of his existence have we any record of his intellectual development. The tombs of ancient Egypt tell a story rich in content of the development of his culture. Dynasties have passed. New dynasties have been born and, in their turn, have gone into the oblivion of the present and the history of the past. Each had its own culture and in each we find the birth of new ideas, of new concepts in government, in literature, and in the arts. In those who lived a thousand years ago, history reveals a type of being intrinsically but little different from ourselves. We could, today, know these men of olden times as friends if they were among us. Their conveniences of life were different from ours, but the elements which made for happiness and sorrow were the same. Even in the subtleties of sarcasm and repartee, as well as in the nobilities of life and the ideals thereof, do we find people like unto ourselves. However, in spite of this similarity in so many lines of development, we find, in history, hardly a vestige of that great realm of natural philosophy, of science, which has revolutionized so many things in the life of our generation.

As we look at the sun shining today much as it probably shone a thousand million years ago, it is indeed strange to contemplate that in those eons of the ancient past, when it stood alone, and when none of the conditions which we know on earth today existed either in itself or in its vicinity, its

substance even then possessed the potentialities necessary to provide for the existence and development of a state of affairs such as pertains to our civilization today. The potential fertility of that formless age is still more amazing when we realize how small a part these marvels of today would have played in the life of the sun itself had not that primeval catastrophe brought our world into existence. In those bygone days, when all the laws of physics confined their attention to conditions in the stars, what strange forethought provided for the possibility that in a new age, born by accident, they should provide also for the existence of a dynamo, an X-ray machine, an automobile, and the various other peaceful, and alas non-peaceful, adjuncts of our civilization. It is, moreover, a matter for wondrous contemplation that it has been reserved for our own generation, and indeed for many who live today, to see the birth of ninety per cent. of those things which represent the whole history of achievement in the world of science.

Comparison of Ancient, Medieval, and Modern Philosophy in Science. Before the time of Galileo, three hundred years ago, science, as we know it today, was practically non-existent. It was not that mankind did not interest itself in the phenomena of nature, but rather that in seeking to understand them it invoked dogmas and superstitious beliefs drawn from nowhere but from man's own consciousness as to how some deity, or other mythological potentate, might be supposed to have decreed the way in which things should happen. To question such beliefs was heresy, to deny them was blasphemy. The difference between the medieval methods of thought and those of modern times may be illustrated by the supposed methods of approach of two individuals, A and B, who seek to investigate and understand the inner workings of one of our governmental departments. Let us suppose that the department is concerned with the regulation of matters pertaining to automobiles. A, symbolic of the ancient spirit, says: "I will not look into the offices of this department and see what is going on, for that would be disrespectful to George Washington, to Benjamin Franklin, and to the other esteemed fathers of our nation, whose initial acts and creation of the nation were ultimately responsible for there being a Bureau

of Automobiles at all. No, I will meditate upon the characters of these noble men and I will draw some conclusions as to the only kind of a Bureau of Automobiles which could have arisen in such a manner as they would have foreseen and as would be consistent with their policies and ideals as to what a Bureau of Automobiles should be like." Somebody may raise with A the difficulty that there were no automobiles at the time of George Washington, and that it is unsafe to draw, from the history of the behavior of the father of our country with regard to matters 150 years ago, conclusions as to what might have been behind his mind in the matter of looking into the future so as to provide for what a Bureau of Automobiles would be like, if and when automobiles were invented and a Bureau to control them was brought into existence. If A had the attitude and faith of medieval times, he would probably say that he was quite sure that George Washington and his contemporaries would never have permitted a nation to arise in such a manner that there could have developed out of it a Bureau of Automobiles which did not function according to the way in which he, A, thinks that George Washington would have thought that it ought to have functioned. Now the attitude of B, symbolic of the modern spirit, is quite different. He quite disrepectfully walks into the Bureau of Automobiles to see what is going on. He sees a great complexity of regulations, former rulings, prosecutions, orders, and what not. He proceeds carefully and without preconceived ideas to find out what he hopes will be the relatively few guiding principles upon which all these decisions are based. He wishes to find what we may call the code of regulations, so that from it he may deduce the consequences of all the special cases without remembering them as a set of disconnected examples of the activities of the office. Both A and B seek a code; but while A starts off by assuming that it must be what he thinks somebody else thinks it ought to be, and then tries to force the decisions and activities into this arbitrary code, B seeks to find what the code actually is. B is symbolic of the spirit of science which came into evidence at the time of Newton and Galileo, a spirit which has persisted ever since, and which is representative of the spirit of the physicist of today.

I fear that you will take a malicious delight in my words, and to the discredit of my profession, if I say that the purpose of the mathematical physicist is to take a simple thing and make it complicated; and yet that is what it is. The many disgruntled students who have had a grievance against the subject from time immemorial will say that all along they have felt that there was something like that in it. Let me ask you to control your wrath for a few minutes, however, while I try to explain what I mean and justify my existence.

When Newton discovered what we call the law of gravitation, what he really discovered was that a planet moves in relation to the sun in such a manner that its acceleration is directed at each instant towards that body and is, in amount, inversely proportional to the square of its distance therefrom. In this statement, I have not even used the words "force" or "gravitation." Their function is purely incidental. They are for the convenience of the dictionary. It matters little if there be some lay members of the audience to whom this statement itself presents difficulties as to its meaning. All I wish to say is that the statement is simple and relatively brief. The simple fact embodied in it was not the thing that was directly observed in the heavens. What was observed was the apparently very complicated state of motion of the planets; and the strength of Newton's discovery lay in the realization that these terribly complicated motions were merely a complicated result of this very simple fact. By regarding them as such we feel that we understand them much better. The ancient and medieval philosophers would have sought some reason for the simple starting point and would have been unsatisfied if they had failed to find a reason in the decrees of the gods. As a matter of fact, they would never have been successful in hitting upon this starting point, because they never would have sought it in the manner in which it was sought by Newton. They would have sought all of their inspiration in the voices of the gods rather than in the heavens themselves.

Starting with the simple fact embodied in Newton's law, we can, by calculation, predict the subsequent course of the heavenly bodies in terms of their state now. We can predict the orbits of the planets, the times of occurrence of eclipses,

the times of return of comets, and a hundred other things. Newton found a simple thing of which the story of the heavens was merely a complication. He took a simple thing and made it complicated, so that this complexity, born before our eyes from simplicity, terrified us no longer. And so, the physicist today seeks to find simple laws in whose offspring he may find a complexity which is the complexity of nature.

When I tap this table, I create in it vibrations which are so complicated that no mathematician on earth could calculate them. Yet, I feel that I understand them because I believe they are the outcome of a very simple law concerned with the way in which a little element of volume of the substance of the table moves in relation to the motions and strains in the other parts of the table around it. I can work out in detail cases of the same general character as that presented by the vibrations of the table, and so I am not disturbed by the fact that the mathematics of the table itself may be too difficult for me. There is great *complexity* for me in the vibrations of the table, but no *perplexity*. You can make the physicist work hard and perhaps lead him to giving up the job by presenting him with problems of sufficiently great complexity; but complexity does not make him mentally unhappy. It is perplexity which drives him to distraction until the perplexity has been resolved by his finding those fundamental (and he always hopes simple) laws which govern the phenomena in which he is interested.

The Rapid Development Since Newton and Galileo. Following Newton there was a period of great activity, and then a period of stagnation which lasted until about a century ago, when that embryo which was electricity developed into active life. In its adolescent state as nurtured by Franklin, by Ampere, by Henry, and by Faraday, it was but a plaything of the philosopher, a plaything so trivial to the layman as to cause the greatest statesmen of the time to scoff at its value, to demand wherein lay its practical use, and at best to accept it with patient tolerance, as the output of the minds of the men of science who, while childlike in many things, were obviously such profound and clever people that if one did not let them do these harmless things there was a liability of their getting into serious mischief and disturbing the tranquility of

mankind. In but a few decades, however, these phenomena of such apparently minor significance grew into the science of electrical engineering and formed the basis of all those marvels which we know in that field today.

It is strange to contemplate the fact that a hundred and fifty years ago little more was known of the science of electricity than the fact that if a rod of suitable material is rubbed with the skin of a cat it will acquire the power to pick up small pieces of paper, and, if viewed in the dark, will be found to emit a blue glow. One can hardly imagine a set of phenomena more vulnerable to the scoffer; for rods and cats have been the stock paraphernalia of witches from time immemorial—the blue light visible in the dark adds no particular prestige to the phenomena. And then, we find that these things will not reveal themselves in the presence of water. Now, we should say that water destroys the electrical insulation, but the scoffer who had heard so much of the fundamentality of that triumvirate, "earth, fire and water," might find ample wherewithal to whet his sarcasm, and even though he should admit the reality of the phenomena themselves, he might well attack them on the basis of their futility; for it would appear that if all the rods in the world were rubbed with the skins of all the cats, the most that we might hope to accomplish would be the raising of a small weight of totally insignificant amount. And yet, on this earth at that very time there existed, and within the reach of man, the wherewithal to make a dynamo.

To one who contemplates the enormous manifestations of electrical power today, it seems almost inconceivable that all of these potentialities could have remained dormant for the whole period of man's civilization.

Following this great period which saw the development of what we may call the science of electrical machinery, once more there was a period of stagnation, a period in which would-be Ph.D.'s went about like roaring lions seeking something to measure. Only by measuring it more carefully than anybody else could the effort be elevated to a rank sufficient to acquire a thesis. Even some of the greatest leaders of thought of the day concluded that science was ended; for that which remained to be known had obviously to do with the structure of atoms and molecules and so lay in a realm of

things so small and light as apparently to lie beyond the realm of man's experimental powers.

And then came the discovery of the electron, the ultimate particle of negative electricity, soon to be followed by the discovery of what was then thought to be the fundamental particle of positive electricity—the proton. The peculiar characteristics inherent in the fact that these particles were electrically charged and were found in nature moving frequently with high velocities rendered them susceptible to measurement in a manner which would have seemed inconceivable to the physicist of 150 years ago. This period was most fertile in the revelation of new phenomena. It gave us the X-rays, it gave us radioactivity, the science in which we find that atoms like radium can occasionally blow up with the emission of particles of high energy, of three or more different kinds. In this period we learned of the photoelectric effect, the emission of electrons from metals under the influence of light. We learned of the thermionic effect, the emission of electrons from metals when heated to a high temperature. And we learned of many effects having to do with these small particles and subgrained phenomena in which the man of science then became encouraged to hope that by their study he might delve more deeply into the secrets underlying the structure of nature and of the atom in particular.

Pure Science the Mother of Utilitarian Science. It is a significant fact that practically all of these discoveries and practically all of the initial investigations concerning them presented at the time characteristics which seemed to place them in the category of complete uselessness as regards any of the practical affairs of life. Yet, as we all know, radioactivity and X-rays later played an important part in the treatment of disease. The further study of X-rays has provided us with one of the most sensitive methods of testing the structural characteristics of materials. The thermionic effect —the emission of electrons from hot bodies—has given us the radio tube and a great number of related appliances which play a part in an enormous number of utilitarian activities today. They are used not only in wireless telegraphy and telephony, but as improving adjuncts to the older systems of communication by wire and cable. They are used in sound

recording, which has revolutionized the movie industry. They are used in all sorts of devices in connection with elements of engineering control, to say nothing of the fundamental part which they play in a very large proportion of all scientific investigations.

The photoelectric effect has given us, in addition to television, all sorts of other devices, such as those concerned with measurement of the intensity of light for photography, burglar alarms, automatic traffic signals, and a hundred other things. One could spend many evenings in the enumeration of all the practical appliances with which the world has been enriched as the result of those investigations which were initially performed during the last decade of the last century and the first two decades of the present century.

Atomic Structure and the Development of Abstract Philosophy in Science. As I have already implied, when first the discoveries of the modern era revealed themselves, the man of science was filled with the hope that now had come the time when he might expect to fathom the principles underlying the structure of the atom. In these days when the streets glow with varied colors at night, it is a matter of common experience that gases subjected to the electric discharge can be made to radiate all sorts of different kinds of light. The more detailed study of this light by means of a spectroscope which analyzes it into what we may call its pure colors reveals a rich story of harmony of relationships, which ever since its discovery has presented to the physicist a challenge for its explanation. Many phenomena of matter are concerned with the actions of matter in bulk, but it soon became evident that the story of this light was the story of something characteristic of the atom itself. However, while we have long since given up the habits of the ancients to seek in the characteristics of their supposed deities the reasons for the phenomena of nature, and while since Newton we have let the facts themselves guide us in the choice of our theories, the philosophy which had evolved was one which seemed to present the greatest difficulties in harmonizing in itself the kind of behavior suggested for the atom by the light which it emitted. Once more we seemed to have reached the end of discovery, the feeling at this time

being rather to the effect that it seemed obviously impossible to know everything that could be going on in the atom, and that probably if we did know it all we should understand why things behaved as they did. However, nature would not let man sleep for long. She did not allow him to plead the mere excuse of complexity for failing to solve his problems, for she began to reveal phenomena in which the atom itself was not directly involved, phenomena in which there could be no excuse on the basis of complexity, but which yet left the poor harassed physicist in a state of much perplexity. Thus, light itself, when traveling in free space, behaved in some ways as though it were constituted of spreading waves, while in other respects it behaved more like a stream of bullets in which the whole energy of a bullet was concentrated in the bullet itself until it had performed its function by hitting something. Light coming to us today from those far off stars which sent out their message long before the first man was born, that light, if composed of spreading waves, would be very feeble in all respects on its arrival here. Yet in the most fundamental respect we find that it is as strong as if it came from a star mounted on the top of City Hall, if you will excuse the slight difficulties involved in the matter of accommodation. A star acting like an emitter of bullets of light sends into our telescopes very *few* bullets compared with what it would send if we were nearer to it, but each bullet which arrives knows nothing of its journey and is as strong in action as when it started. To have to consider as of the nature of bullets this light which science had regarded for the last three hundred years as composed of spreading waves was a matter of great perplexity for the man of science, particularly when in some connections it was still necessary for him to retain the wave picture. However, he was loth to make a change in his views and his pictures of what was meant by the reality of things, and it was consequently necessary for nature to put on the thumbscrews a little more tightly. This she did by presenting him with a new set of phenomena in which electrons, which he had always regarded as possessing the most fundamental elements of respectability in the matter of being particles, must in some of their actions be regarded as behaving like waves. This was the last straw, and it became necessary for

the physicist to remould the whole of his concepts with regard to what should be considered an explanation of things in science. Our whole concept as to the meaning of reality became changed in such a manner as to embrace the old philosophy within a new form, richer and far more fertile in content. Many of the elements which have presented points of difficulty in understanding atomic phenomena were seen to be elements of an ultimately irrelevant character; and when we were relieved of them the harmonization of the new discoveries went on at such a rapid pace that in ten years physics did more setting of its house in order than had ever been done before in a period ten times that length when corresponding new fields of the science had opened up.

Again we seemed to come to the end of things. One idea had survived the complete revolution in thought to which I have referred. This idea was concerned with the fact that the atom is composed of a central part, the nucleus, which contains practically all of its mass and whose structure determines the nature of the element, while around this nucleus is to be found a more loosely bound structure of electrons. The story of the outer structure is the story of the atom's emission of light, of its emission of X-rays, and of its power to combine with other atoms. The outer structure, in fact, contains the story of chemistry. However, the nucleus is the thing which determines the difference between the elements. Copper differs from iron because the nucleus of its atom differs from the nucleus of the atom of iron. All of our evidence was in favor of the view that the nucleus is a very closely bound structure, and there was a time when it seemed hopeless to be able to break it up by artificial means and find out how it is put together. However, within recent years, methods of endowing charged particles with very high energy have been discovered, so that it has been possible to fathom even the nuclei and disturb the tranquility of even these most firmly bound citadels of nature. Not only can we realize the age-old dream of the alchemist, the transmutation of the elements, but we can disturb the nuclei by bombardment in such a manner as to produce nuclei, and so atoms, not found in nature. They are not found because they are unstable. They last for a short time and then break down into something else, so that

had nature evolved them she soon would have lost them. These atoms have characteristics similar to radium, which, as I have remarked, is continually disintegrating and which only continues to exist upon earth because it is reproduced by something else as fast as it is destroyed. These new radioactive products which have come into existence during the last few years have a variety of characteristics which make them of particular use in therapeutic work. Radium disintegrates at such a rate that any given amount of it disappears to the extent of fifty per cent. in two thousand years. Some of these artificially prepared radioactive materials have very much shorter lives and this renders it possible to do with them certain things which would be impossible with radium. Thus, if one should inject ordinary radium into the blood stream for some beneficial purpose, even if it should perform that purpose, it would then proceed to devote the remainder of its activities to killing the patient. Since the radium is only half dead in two thousand years, the patient would be liable to lose out. With a radioactive material of half life of a few days or even a few hours, the good work may be performed and the evil work omitted. Then again, if we wish to know how long it takes for the element sodium, for example, to be transferred from one part of the body to another through the blood stream, it is possible to inject a little of the radioactive sodium into the blood; and by means of sensitive devices which can detect it through its radiations, find just where it goes and how long it takes to get there.

In connection with these investigations upon the nucleus of the atom, the family of fundamental particles has been further increased. There was a time when we thought that there were only two such particles, the proton and the electron, and that all the architecture of nature was built out of them. Truly that was a great responsibility for two such fundamental bricks. Now we know that there is a positive electron which is just like the negative electron except that it is oppositely charged. We know that there exists a neutral particle, a neutron, which has the curious property of passing easily through lead and being stopped readily by water. It is one of the most therapeutically active and dangerous of the radiations. There is a particle intermediate in mass between the

electron and the proton, a particle so young in the family of particles that it has hardly yet been christened, and physicists are quarreling over its name. There are photons, the particles responsible for light and X-rays. Finally, there is a mysterious kind of a ghost, called a neutrino, which has never been directly observed in the sense in which the experimentalist understands that term, but whose existence in the atom has been inferred on the same kind of evidence as that which, when mysterious things happen in an organization, causes one to say "Somebody must have been at the bottom of that."

Why Does Discovery So Frequently Appear to Have Come to an End? The Dangers of Common Sense. I have had occasion to refer to several periods in the growth of science when progress seems to have come to an end with no hope of further development. Then something happened, and all was once more alive. What is the reason for this condition of affairs? I think the reason is not far to seek. When a new set of phenomena reveal themselves, the man of science seeks to correlate these phenomena in the form of a theory. Sometimes in the formulation of the postulates of the theory there are details which are artificial to him, there are starting points which seem unreal, so the theory is at first abstract and few understand it. To the layman, in fact, some of the postulates of the theory may seem to constitute nonsense. However, the working purpose of the theory is not primarily to give pictorial satisfaction to the lay mind but rather, as I have already remarked in connection with gravitation, to correlate the phenomena which it concerns. The theory is a means by which, through the process of saying few things, we may deduce many as a consequence. Having moulded the theory so as to comprehend all of the newly discovered phenomena, the theory of itself starts to predict other phenomena; and it is well worth while to look for these phenomena. And so there comes a period in which science devotes itself to verifying the predictions of the theory and tracing its consequences to their limit. In the olden days, this took a long time. In modern times it has taken increasingly shorter and shorter periods, as the would-be Ph.D.'s and their mentors stand around eager to measure anything which comes on the horizon

for measurement. In any case, before very long everything that the theory suggests as an object of measurement has been measured and every phenomenon which it has predicted has been found, if it is a good theory. By this time, the theory has acquired a good deal of prestige. It has been responsible for suggesting many new discoveries. Those postulates and premises in the theory which were a little repugnant to us in the beginning have become more natural and reasonable in our eyes. They sit well in the realm peopled by the interesting phenomena they have brought into it. They, the nonsense of yesterday, have become the commonsense of today. And now, the theory having grown old in a useful life, becomes rather like a conventional old gentleman who, radical in his youth, has become conservative with age, who now hates to see anything other than what he has been accustomed to, and who is able to exert great influence in virtue of the respect in which he is held through his good deeds of the past. Radical in his youth, he has become extremely conservative in his own radicalness. The old theory, having in its active youth exhausted all of its potentialities in saying: "Do this, and thou shalt find that," now starts to adopt a negative attitude in saying: "It is not worth while to do such and such, because it is guaranteed beforehand that you will find nothing if you do." "To find anything," says the old theory, "would constitute nonsense in my creed, and would be very humiliating to me. It would be contrary to common sense, the new common sense of my era." And so the old theory now becomes as declamatic in saying that certain things are impossible, as before it was declamatic in holding that other things must occur. It is for this reason that in the autumn of the life of some far-reaching theory of physical phenomena, science seems to have come to an end. The phenomena divide themselves into two categories; first, those which are known or are such obvious consequences of the known facts, or of the theory correlating them, that it does not seem worth while to investigate them further; and, secondly, those which look as though they are probably embraced by the theory but in a form beset with such complications of calculation as would render the dissection of their whole story beyond the power of man. As a result of

this, there seems nothing more to do; research seems to have come to an end and science to be dead. Then some irrepressible individual discovers a phenomenon which goes violently contrary to the theory. After due castigation by the old gentlemen, in the form of the devotees of the theory, the results of the young upstart are confirmed by others and have to be accepted. The theory must then be remoulded or possibly completely rebuilt so as to include the new facts. In this rebuilt theory, much that before seemed impossible is now rendered likely. Suggestions which would have been dismissed as impossible with the briefest thought in the light of the old theory, have a reasonable place in the realm of possibilities of the new theory. And again common sense grumbles at having to readjust itself to a new form. In his proper domain, common sense is a counselor of priceless value; and it is because he justly inspires such confidence in that domain that he becomes the most dangerous of deceivers of those who seek his guidance outside of it. For "common sense" seeks to pin all thoughts of the new to the fabric of the old, and so, ofttimes, it distorts the meaning of the new by destroying that form which was inherent in its own right, and for no purpose other than to fit it to a pattern in which it has no place. The result is a bizarre and shapeless thing out of harmony with the form into which it has been forced, and out of harmony with the form which was its own. Common sense in natural philosophy repatterns itself from age to age. At each stage of its development it seeks to generalize the ideas born of the experience of the immediate past and to weld them into bonds which sometimes restrain the future. Thus, the breeders of error in the epoch to come are sometimes the truths of the days which have gone.

I think with some regret of many experiments which in my youth occurred to me as worthy of being tried, but which on further meditation I failed to perform because in terms of common sense, as exemplified by the theories of the day, they seemed destined to give trivial results. Sometimes I wish that I had not thought so much and had done the experiments in any case, for in later years many of these experiments have been done and have given results of value, results which are harmonious in the frame of thought of today, however

strange they would have appeared in the light of our concepts of thirty years ago. And so, today I am apt to be chary in discouraging any experiment which lies near the boundary between the known and the unknown, even though the known territory may claim it as a citizen and may claim further to predict for it a result which, if found, would be trivial and of such a nature as to render the experiment unworthy of performance. If a proposed experiment lies far within the boundaries of established facts and theories and would, according to their diagnosis, give a perfectly clear-cut result, I do not deem it worth while to do the experiment. If someone comes with a scheme for perpetual motion involving the utilization of ordinary machinery and the like, I give him but little attention. If he claims that, since the experiment has never been done, one cannot predict with surety what result it would give, I have to agree with him, as I would also agree with him if he claimed that the sun might not rise tomorrow morning. I then answer to the effect that while no one can predict the result of an untried experiment, the *chance* of his perpetual motion experiment giving a valuable result is, in my judgment, so small that we should not be justified in discontinuing the other experiments which we are doing in its favor. If, however, I am confronted with an experiment which lies near the horizon between the known and the unknown, I prefer to give it the benefit of the doubt; for it is such boundary dwellers as these who establish connection with the knowledge of today and that of tomorrow.

The Future of Discovery. Naturally, it is of interest to meditate upon what the future may reveal in the way of scientific discovery; and here, as we peer beyond the horizon of the known, the mist becomes more and more dense so that we cannot hope to look very far ahead with any certainty. We must proceed step by step in that hazy territory and clear the fog from around us as we proceed, if we would advance with reasonable certainty. Of course, it is possible to vision almost anything. You might prophesy any fantastic discoveries for the future, and a mathematical physicist would have but little difficulty in constructing for them a frame of harmonization which, if they ultimately existed, would represent them as consistent with the criteria of a form of common

sense in which we should ultimately be willing to accept them. Indeed, many of the things which we accept complacently today would be ranked among the most remarkable of all miracles, had the man of science not adorned them with a framework to remove them from the realm of the occult. I sit in my study today and I hear the voice of a man in England, or in Germany, or in Russia. If, a hundred years ago, I had heard that voice, I should have said that a miracle had occurred. I should have regarded the miracle with no less wonder had it been necessary for me to put myself into some sensitive state in order to hear the voice. Had somebody told me that I really did not hear the actual voice, but that the voice had been absorbed in space in Germany and had been somehow or other precipitated again out of space in Philadelphia, I should not have been less astonished. In fact, this extra and mysterious mechanism would only have filled me with greater wonder. Today, when I hear that voice in Berlin, I remove the mystery by the interpolation of all sorts of extraneous apparatus and theories. These seem to remove the ultimate fact from the realm of miracles; but I hear the voice, nevertheless, and in the last analysis I doubt whether the extraneous explanations and devices which put me into a sensitive state remove from my thoughts the concept of a miracle except to the extent that they busy my brains with so many things to think about that I have no time to be astonished at anything.

And so, peering through that boundary of haze into the future, I am not prepared to deny the possibility of anything. In the realm into which my mental vision can penetrate with reasonable clarity, however, I see, in the first place, the rounding up of the story of the interrelation of the atoms one to another. Looking a little more deeply into the haze, I seem to see a remoulding of our thought structure in relation to atomic phenomena in such a manner as to reveal more clearly the reason for the existence of such atoms as we have and the reason for the relations which exist between them. I would hope to see more in detail the mathematical blood relationship which exists between those fundamental bricks of atomic structure, the electron, the proton, the mesotron, the neutron, the neutrino, and the photon. I would hope

to see harmonized in greater unity phenomena having to do with matter and radiant energy.

One of the most astonishing discoveries of modern times is the interrelation between mass and energy. Sometimes this phenomenon is spoken of as the conversion of matter into energy, or of energy into matter. I have some criticism of the fundamentality of that terminology, but of that I will not speak. The fact remains that it is by this process that the sun maintains its continual output of heat, an output so great that were the sun no more than a cooling sphere it would have cooled to ordinary temperatures within historic times. The sun provides its heat by a kind of burning of its substance; but this burning is not of the ordinary kind in which, as the result of it, the same elements are to be found in other forms after the combustion as before. The sun's burning is one in which its very atoms disappear as such, or change into others containing less energy, with the result that the surplus energy given out in this way provides for the sun's radiation. The amount of energy rendered available by the annihilation of matter in this manner is enormous. The complete annihilation of the matter in a single drop of water would produce enough energy to supply 200 horse power for a year. These processes of annihilation and creation of matter can now be observed even in laboratory experiments, and particularly in those concerned with cosmic-rays. I am aware of the arguments which render risky, and some would say even worse than risky, the hope that a way may ultimately be found to convert matter into energy in usable amounts in the laboratory. Yet, I realize that the elements concerned and the principles involved lie rather near that border line of knowledge of which I have spoken; and while I will not guarantee to you that energy will ultimately be made available in this manner, I certainly will not guarantee to you that it will not.

There is always a long drawn out period in putting scientific discoveries to practical use. Radio tubes are still being devised and improved today, while the fundamental discoveries with regard to them were made 25 or more years ago. The photoelectric effect was discovered long before the beginning of the century, but television is even now only

emerging from the experiment state. And so we may expect quite a long period of development in which the discoveries of the last few decades continue to bear fruit in the practical and humanitarian affairs of life. Some of these developments will simply result from perfection in technique in respect of fundamentals whose general nature is already known, as in the case of television, where the continual improvement of fluorescent screens, of methods of getting finely focussed beams of electrons, and a hundred other things, conspire together in their development to produce an ultimate result whose perfection becomes inconceivable when viewed from the possibilities available two or three decades ago, in spite of the fact that the fundamental phenomena themselves were even at that time well known.

Undoubtedly one of the greatest fields for development of science lies in the borderland region between physics and biology. Any science such as biology starts with the observation of the more obvious facts and with correlation of the relationships between them. In the process of this complete correlation, we find it necessary to look deeper in order to complete the correlation. And so, having started with the animals and plants themselves, we dissect them to learn more of their structure, and we invent microscopes from which we learn that from a more fine-grained standpoint they are composed of cells. We examine the cells and find that they have a structure, a nucleus and various other elements. We study the behavior of these things, and the knowledge gained reflects back upon the more coarse-grained phenomena and enables us to understand them better. We see the details in the division of the cells, and we understand better the growth of the animal. And as we proceed in this way, working down more and more towards the fundamentals, we approach closer and closer to, and make more and more use of, the principles of physics, for physics itself is the science of the fundamentals of all matter. Even the crude elements of physics serve to solve many of the relatively fine-grained problems of biology. Thus, a knowledge of relatively crude physical laws sheds much light upon the phenomenon of the motion of certain cellular organisms in electric fields and the dependence of that motion upon size. A biologist once

brought to me what appeared to be a remarkable phenomenon concerned with the difference between dead and living matter and having to do with the fact that certain spores showed different properties in a certain electrical experiment according as they were alive or dead. On looking into the matter closely I found that the difference in properties in question were all attributable to mere difference in size, so that the question of the organisms being alive or dead was only involved to the extent that they were of different sizes in the two cases.

While, however, the coarse-grained laws of physics serve many useful purposes in correlating the phenomena of biology, the more fine-grained phenomena of atomic structure stand ultimately as a background to all phenomena concerned, and the utilization of the great knowledge obtained in these matters in recent years is still a problem of the future. It is not a problem without hope, however, and it seems likely that the understanding of certain vital phenomena which have not yielded to the application of biological principles alone, or even to those principles supplemented by the coarse-grained laws of chemistry and physics, will show a greater response to understanding when viewed in the light of the knowledge which we have gained in recent years as to the elements which control the structure and equilibrium of molecular entities. I refer particularly to those very large organic molecules which seem to play a part in such diseases as influenza, and possibly in cancer, molecules in whose properties the line of transition between dead and living matter becomes ill-defined.

The difficulty in all applications of physics to practical problems results from the complexity of the phenomena presented. When the physicist works in his own domain he chooses simple problems in terms of which to illustrate his principles and exhibit his skill. Nature has been rather kind to him in presenting him with one or two relatively simple problems for the exercise of his efforts. The problem of the hydrogen atom, a very important atom, has been the joy of mathematical physicists in all stages of development of the subject. It is simple enough to handle fairly completely and yet can develop characteristics of complexity which are sufficiently great to be alluring in taxing the

mathematician to solve them, but not so great as to discourage him. As we proceed to the heavier atoms the difficulties become increasingly acute. The problems of interaction of atoms with atoms and of molecules with molecules present complexities of even more serious kinds, and yet nature has a strange way of rejuvenating its simplicity in various stages of its complexity, so that the problem, for example, of two atoms revolving around each other, while at first sight of a degree of complexity to baffle the powers of the greatest mathematician for a solution, rejuvenates, nevertheless, in approximate form a kind of simplicity in which the spirit of the fundamental laws is still reflected. Even when we come to such complex aggregates of atoms as are involved in crystal structures we are not powerless to proceed. And so, I think that we should not be discouraged in the hope that our greater understanding of atomic matters acquired in the last few decades will prove of fundamental service in enabling us to understand and possibly control the behavior of those molecular aggregates of which I have spoken and which stand as strange links between living and dead matter. Perplexity in the problem is not far from being removed and let us hope that complexity will ultimately be defeated.

Scientific Progress and Human Happiness: Happiness a Condition of Change for the Better. I suppose there is no age which has not known the question, "Are we happier than those who have gone before?" It certainly seems as though in our age we should be. The laborer of today has at his disposal conveniences which no king possessed two hundred years ago. I think it would be safe to say that if King Solomon could suddenly have had installed in his house an oil furnace, a cooling system, electric light, and a telephone with the other end at the residence of the Queen of Sheba—if he could have gone careering through the streets of Jerusalem at fifty miles an hour in an automobile, he would probably have been regarded by the rest of the world as renowned for possessions to an extent far beyond even the renown recorded to his credit in Holy Writ. Today, even the humblest artisan is possessed of conveniences which, seemingly, would have outshone all the luxuries of the

world of ancient times. Yet, he who possesses these things today is often an unhappy and disgruntled person, with a grievance against something or somebody as his main source of mental exercise.

When I was young, I lived in England in a cold house. Nobody had a furnace. You lighted a fire in the room you intended to occupy and got as near to it as possible to keep warm. When I first came to this country, and following a period in which I was almost suffocated by the temperatures of the houses, I came to a realization of what a marvelous thing it was to have heat everywhere and hot water always. However, I have now grown to a sort of placid contentment in which my reactions are never those of exhilaration, but only of occasional annoyance when the heating plant goes wrong. I don't think I am any happier in the thermal sense than I was in England, although if you took me back there and put me in the aforesaid cold house, I should howl as loudly as any native born citizen of Washington, D. C. In a sense, however, I have lost something; because, never actually unhappy in the cold, I was distinctly happy in the emphatic sense when, on coming down from a cold bedroom in the morning, I found a nice fire to stand in front of. The fact is that happiness for the most part results from *change* of conditions, and you get just as much change by coming from something worse to your present state as you do by going from your present state to something better.

In the good old days in England, there were no telephones, or none to speak of. If you wanted to communicate with anybody, there were, in general, two possibilities. The first was to change your mind and decide that you really didn't—the second was to send a post card. The first was really more frequently used. The possession of a telephone gives enormous joy for a short period. Then it becomes a matter of course, and then it becomes a means of doing a lot of things you really need not do, and having a lot of other people do things to you that you would rather they wouldn't do.

When a young man in England, I had no secretary, and nobody to type my letters. There would have been nobody to type this speech, so there probably wouldn't have been

any speech. Think what a lot of effort that would have saved me—and you.

I remember the enormous thrill I got on first owning an automobile. My only thrills now are those of anger when a battery goes dry or a tire blows out. As a matter of fact, I do not see the country any more because, being in the automobile, I feel I must drive fast, as otherwise what is the good of having an eighty horse power engine? And, indeed, if I don't drive fast, I hear honks from behind urging me on. If my wheels deviate one degree from their proper course, I shall be off the road into the ditch in just about six seconds at sixty miles an hour. I dare not look anywhere but where I am going.

The great advances in science have added much to the elimination of the positive sources of *un*happiness, to the elimination of disease, to the elimination of that drudgery which was a necessary accompaniment of man's mere existence, an existence, moreover, which a hundred years ago seemed to have, in most cases, little purpose other than its own perpetuation into another generation. Once science has accomplished this end, however, its further power to give joy is limited by the fact that *positive* happiness of mankind, the kind of state in which man is happier alive than had he not been born, is determined rather by changes in his condition than by that actual condition itself. New potentialities for happiness flash like meteors into the sky of man's existence; but they die and fade ere long except in so far as they provide means apart from themselves for a continuance of the happiness of which they have given potential promise. Science will not provide positive happiness for the lazy. Only in proportion to his labors can man expect to reap enjoyment. Even in the work which is for pleasure, all will not be pleasure. Certain elements of drudgery must be accepted as hurdles in the steeplechase of life's enjoyment. To sit down behind the hurdles is to forego the joys of the chase. Man can find nowhere but in himself the means to secure a continuance of his own happiness. If the world has provided him with all the means that most men crave in the thought that they will bring contentment, forgetful that only in the acquiring of them is the thrill to be found, then, unless he is otherwise

employed, this man, saturated with wealth and the goods of the world, will find himself turning to carpentry or gardening as a hobby and doing so much work at the job that, if he were compelled to do that amount of work for a living, he would be a continual suppliant at the doors of the local trades union to preserve him from the tyranny which compelled him to do it.

Now, I do not think it a lamentable thing that the multi-millionaire should turn to carpentry to earn a living—in the sense that he must do something to make life worth living. I do think, however, that it is very important that he should become a *good* carpenter, for unless he does, he will do little but cut his fingers and rearrange his tools. He can hardly hope to gain a million dollars worth of satisfaction per year out of that. If, however, he decides to become a really good carpenter, he will have a continual sense of enjoyment following from improvement in his work. Indeed, I would state emphatically that in this, as in all things, there is more happiness in the consciousness of improvement than in the finality of attainment.

The Scientific Method in Social Affairs. In these days of economic stress and world turmoil, there are many who wonder whether the methods of science, which have been so successful in winning the battles of the inanimate world, may not be invoked in the social problems of those who inhabit it. I think they may; but in order that they may be successful, it is necessary that all concerned shall take account of the essential special features of the problem; and the start is difficult, because so many are ready to object before that to which they propose to object has been formulated. It is true that the problems presented are very different from those of the inanimate world, and yet I doubt whether the difference is as great as is sometimes supposed. I doubt whether the reorientation of scientific viewpoint necessary is as great as the reorientation which was found necessary in the inanimate world itself in the transition from the methods of thought of a hundred, or even twenty-five, years ago to those rendered necessary by the physics of today. I realize that animate beings are different from inanimate things.

Some time ago we changed the vacuum pumps in the Bartol Foundation because the ones we had had grown obsolete and there were newer and better kinds. Had this been a matter concerned with human affairs, I surmise that on the morning following the change I should have heard the old pumps walking laboriously up the stairs on their creaky joints to my office to lodge a complaint. They would doubtless have pointed out that they had become old, leaky, and decrepit in our service and that to set them aside was a very wicked thing. Even if I should promise to put them on a less important job, they would probably feel insulted and toddle around the laboratory raising trouble with the rest of the apparatus.

I believe that George IV was interested in electrical experiments, and that in London are housed some pieces of apparatus which he used. They are provided with carved protuberances and knobs, and adorned with such ornamentation as seemed necessary to provide them with the dignity suitable for the experiments of a king. Apart from this, they are very old and very useless. However, if they were alive, I imagine they would have much to say as to their dignity and the part which they felt they should play in modern affairs.

Theories, Common Sense, and Social Affairs. It is very customary in the affairs of men to take a word, or statement, which has attained a bad reputation in some connection, and extend the evil reputation to every other connection for which the dictionary will permit the use of the word. One of the words which has obtained this bad reputation is the word "theory." The ancient and medieval philosophers had theories about the universe; and as we have seen, they were formulated without connection with nature, and led men astray in their predictions. People often draw the inference that anyone who starts to think about the fundamentals of matters is to be placed in the category of these philosophers of bygone days, forgetful of the fact that, in the sense in which the word "theory" has meaning today, the premises of a theory are simply short cut methods for expressing the totality of the facts, and are born from the facts themselves.

To state that certain principles are all right in theory but contrary to fact is to state something inconsistent with the very meaning of language itself. In times of great stability, experience is a priceless guide. I have referred to it in connection with scientific matters by the term "common sense." In times of change, however, it is sometimes a dangerous counselor when not controlled by those additional weighings of the special elements concerned, which is the work of him we call "the theorist." It was the common sense of the ancients which made them see the Universe as a box with a river running around the outer edge, the river carrying a boat and the boat carrying the sun. It was common sense which, in the time of Keppler, saw the sun endowed with spokes to which were attached the planets, and which, through the rotation of the sun, drove the planets around in their orbits. It was the common sense of the astronomers in the time of Galileo which made the discovery of the satellites of Jupiter seem nonsense and his discovery of the laws of falling bodies contrary to reason. Common sense is a good servant but a bad master, so beware how you glory too much in "good old horse sense" for, in the last analysis "horse sense" is, in all verity, the kind of sense that a horse has.

Laws of Nature and Social Affairs. In speaking of social matters, one encounters much misconception as to the meaning of what we call a law of nature in science; and there is a tendency to seek in the social field certain laws of such a fundamental character that nothing that man can do can prevent their operation in one and only one way. I often hear the "law of supply and demand" quoted in this category. Now, to my mind, such a law has absolutely no meaning unless one states the conditions, in the form of other laws natural or created by man, according to which it operates. There is in science a very famous law which shows close analogies in this respect. It takes various forms, perhaps the best known of which is the form referred to as the "principle of least action." To cut a long story short, the law is to the effect that natural phenomena take place in such a way as to keep a certain mathematical quantity a minimum. At first sight the statement seems to say something perfectly definite; and indeed, the law originated on theological grounds con-

cerned with the idea that the Creator would not be wasteful. The thing about which the older philosophers were very vague, however, was the question of what it was that was not to be wasted. From a modern standpoint, the mathematical physicist regards it as his business to find what it is that he must talk about in order that, when he demands that it shall not be wasted, that demand shall require the universe to behave as it does. He may then say that the demand that this thing shall not be wasted is another way of stating a law of nature. However, as every mathematical physicist knows, the statement that things shall occur in such a way that a certain quantity shall be a minimum—that there shall be the least waste—is in general subjected to certain additional mathematical conditions; and the requirement that this quantity shall be a minimum reflects itself in entirely different ways upon the ultimate result as regards the phenomena of nature, depending upon the supplementary laws or conditions according to which that law of a minimum is allowed to operate.

There are, in science, certain conditions which are curiously, and even amusingly, analogous to certain conditions in the world of affairs. Thus, an assemblage of gas molecules, such as we have in this room, may be regarded as a highly democratic, or even socialistic, or even communistic assemblage. Their motions are governed according to what we call the laws of chance—the laws of chaos. All the molecules do not have the same velocity, there is a distribution about an average, but the chance of a molecule having more or less than the average energy becomes extremely small if the amount of energy in question is far different from the average. In fact, the mechanism of the process is such that if any particular molecule happens to acquire a very large amount of energy, all the other molecules proceed to bombard it in such a manner as to rob it of its success and try to bring it to the normal level; and if any molecule has very much less than the average, the tendency is for the other molecules to boost it up. The mathematics of chaos shows us that even in this most chaotic of assemblies, in which all effort is towards equalization, the final state of the molecules is one in which they do not all have the same energy.

Another aspect of the story of tendency towards equalization in the case of the gas molecules may be expressed in a manner which I think has meaning also in other fields and is to the effect that any large statistical aggregate of entities has a minimum of useful properties, or in fact, of distinguishing properties, whether they be useful or not. The tendency of the molecules is to dislike differentiation between one another. Moreover, it is rather sad to contemplate that according to the general belief of science the structure of our whole universe is destined ultimately to disappear, and that the laws of science are of such a nature as to cause everything which happens to lead one step nearer to that morose end.

Of course, in working in the purely scientific field one is relieved of the embarrassment which would be involved if the entities whose laws we investigate had the power to complain of our findings. Moreover, the entities do not have the power of free will to inveigle other entities by propaganda into their service and cause them to assume characteristics different from those customary to their nature. In the world of affairs we seem to have to reconcile ourselves to inconsistencies which could not be tolerated in the field of pure science.

Man is a curious creature; and in the aggregate his actions differ from those of the individual. Any group of beings has in some sense the characteristics of an animal, which can be more sensitive, more responsive, less intelligent, more cruel, and more primitive, than any of the individuals of which it is composed. And so this animal which represents groups of men behaves in ways different from its individual members. If as an individual I owe a debt and have the money to pay it, I am not relieved from payment by a plea that my house needs mending, or that if I pay it I shall not be as well off as I was before. A nation, however, as experience has shown, will regard it as a necessary criterion for payment that it shall be no worse off as a result of the payment. If, as an individual, I come to a legal agreement with somebody at a time when it seems that the agreement will be to my benefit, I am not relieved of the responsibilities of the agreement if it should happen to be invoked in circumstances which react to my loss. A nation, however, having

made treaties, will cry to high heaven when they are violated against its interests, but will itself consider the pressure of circumstances as a sufficient justification to violate them itself, if such occasion arises. To bomb civilians in war is a crime, but to bomb a village of savages because some unknown member of the tribe has shot at somebody is but just punishment, or even education.

In a sense, the study of human affairs today is encumbered with hurdles like those which stood as a barrier against the progress of science in the Dark Ages. That which was acceptable as truth in those times was not determined by what nature demanded, but by what certain elements of mankind wished that she should demand. The truth in science was feared as something harboring elements of catastrophe to the life and conduct of man. These fears have long since evaporated and the truth, when taken in its totality, provides in itself for a harmonization of belief and action such as to give no cause for the fears of old. Let us hope that the day may come when man can adjust his ambitions in relation to his philosophy of life in such fashion that all elements of discord and unhappiness may dissolve, and leave only that which is beneficial to mankind. Then will science walk hand in hand with human development as its constant benefactor, as the guardian of its peace, in a universe rich to provide happiness and security for all.

Rubber Coated Electrodes.—Adjustable electrodes that look and feel like rubber pads are the newest gadget to be added to home diathermy machines. With the short wave therapy apparatus already in wide use in hospitals for the treatment of painful diseases, the rubber-coated electrode makes possible safe use of the portable short wave radio set in the home. Patents on the new device are held by the Home Diathermy Co. of New York. The small rubber electrodes may be fastened to any part of the body in the same manner as an ordinary electric heating pad. As the 16 meter radio short-waves come in contact with internal liquids, they turn into heat waves, stimulating circulation in the painful parts and thus removing the poisonous toxins. A glowing sensation of internal warmth results, but the outside of the body remains cool.

R. H. O.

Transatlantic Radio Photos.—Overseas Radiophoto communication took a big step forward recently with the announcement by R.C.A. Communications, Inc. of the commercial inauguration of a new system for the transatlantic transmission of Radiophotos. Use of the new method, which has been several years in development in the R.C.A. laboratories, now makes possible the transmission of pictures and other graphic material in vastly improved detail with transmission speeds at least three times as great as heretofore achieved. The new system overcomes the necessity of transmitting transoceanic radio pictures in a series of dashes, like small pen strokes of varying length. The light and shade values of a picture now come over as a continuous electrical wave, with certain characteristics that vary in exact proportion to the values of light and shadow of the subject. The system is virtually indifferent to the effects of fading or "radio echoes" imposed by atmospheric conditions on long range, short wave radio transmission. It is regarded by technicians as the biggest step forward in the art of overseas radio facsimile since R.C.A. achieved the transatlantic reception of the first crude image in 1923.

R. H. O.

PROGRESS IN METALLURGY: THE SCIENCE OF ALLOYS.*

BY

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The science of alloys is the motorcycle squad leading the parade of progress. From the earliest recorded alloy which appeared in Ur about 4000 B.C. until the present time, man's technological development has been limited by available materials, and the science of alloys, continuously providing new materials, clears the path for progress. The Electrum alloy of Ur made possible the first "modern" coinage system, and the X-ray target alloys of today make possible the latest system for locating the building blocks of organic chemistry in the complicated synthetic molecules. In the interim there has been a constant development of alloys for tools, instruments, machines, and electrical and chemical apparatus. A mere recital of the more common alloys would take more than the period allotted for this discussion, so that we will consider only those of more recent interest and those which best exemplify the principles of the modern science of alloys.

The everyday importance of the science of alloys is obvious, but man is so ready to accept the obvious that its significance is frequently lost. In general, the application of the alloys may be divided into four groups: master alloys for making other alloys, tools, instruments, and finished products for direct consumption. The master alloys are found only in the metallurgical industries. The tools are more apparent in everyday life. Those found in a modern kitchen well illustrate our dependence on alloys. Forks, knives and spoons are made of alloys of silver-copper, copper-nickel, nickel-zinc-copper, chromium-iron, or chromium-nickel-iron. The electric iron may have an iron-silicon-carbon or chromium-copper alloy base, and a nickel-chromium resistance element—all a result of alloy development. The mixing machine carries an

* Presented at a meeting held Thursday December 1, 1938.

electric motor built largely of silicon alloy steel, with a commutator which may be made of a silver-aluminum-copper alloy, and with phosphorus-tin-copper alloy springs. This motor drives a steel alloy shaft with a stainless alloy steel mixing blade—again a tool made possible by alloying.

As to instruments, a thermostat controlling the furnace may be seen in the adjacent hall. This is an assembly of parts made of alloys of copper and zinc, iron and nickel, nickel and copper, phosphorus and copper, and tungsten and silver. It is a measuring instrument of scientific accuracy, functioning as a controller and available to every household because of the development of alloys. And the telephone on the table is an instrument of a hundred alloys, each of which has been developed because of specific characteristics enabling it to fulfill a particular function.

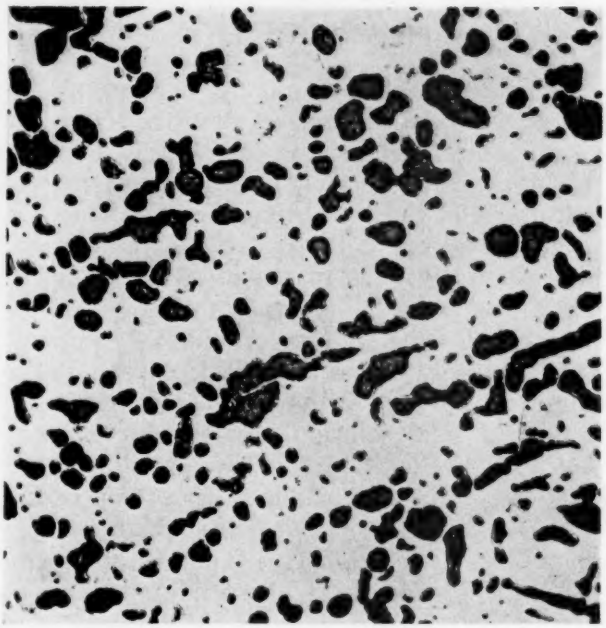
Consider finished articles of direct use. The kitchen sink may be an alloy of nickel and copper or of chromium, nickel and iron. The faucets are alloys of copper, tin, lead and zinc. The stove may carry a high chromium-iron alloy top, and the refrigerator may be fitted with alloys of aluminum, copper, zinc and silicon, as well as alloys of iron nickel and chromium. If we step from the kitchen to the garage, we find the automobile which is built of some 125 different steel alloys and a host of non-ferrous alloys.

Thus it will be seen that the development of alloys is a matter of real importance in our everyday lives, and that the science of alloys has done much to increase our standard of living.

The question may well be asked "What is an alloy?" and indeed a truly satisfactory definition is difficult. In the broadest definition, a metallic alloy would simply comprise one metallic element plus one or more other elements mixed with sufficient intimacy. An alloy may be more effectively defined as an intimate mixture of one metallic element and one or more other elements which are or at one time have been mutually dissolved. Strictly speaking, this may not take into account synthetic mixtures of powders which have been compacted, but even in this case it may be considered that the compacting may effect surface bonding akin to alloying. The mutual solubility above mentioned may have

taken place only in the liquid phase, and after freezing there may be almost no mutual solubility. Still the intimate mixture which results is an alloy. Or again, powdered metals may be pressed together and sintered so that mutual solubility takes place along their borders, which solid solution may or may not continue to exist at room temperature. At any rate the intimate mixture is still an alloy. Broadly, alloys may

FIG. 1.



Emulsion type alloy. Copper containing 24 per cent. lead-unetched. 100 ×.

be divided into four structural groups: solid emulsion type, solid solution type, precipitation type, and eutectic type.

The alloys representing the solid emulsion type are difficult to prepare if optimum properties are required. There is little need to describe such alloys, except to state that the object is to provide a mixture in the solid phase which is the analogue of an emulsion in the liquid phase. The alloys of copper and lead (Fig. 1), and of iron and lead are illustrative.

Copper-lead alloys may contain as much as 55 per cent. lead, although solubility is limited even in the liquid phase. Alloys containing up to this amount of lead are used for bearings, where the presence of the lead produces low frictional resistance. Compacted metal powders may approach this type. Lead in the iron-base alloys is a matter of very recent development, and the frozen emulsion which will contain some 0.2 per cent. lead is prepared with practically no liquid solubility. The object of adding the lead in this case is to provide small films of lead or lead compounds so distributed in the final rolled product that machinability is markedly increased.

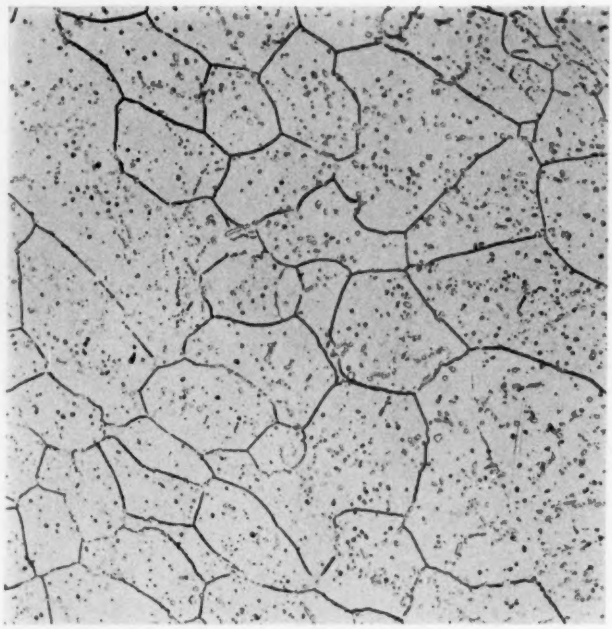
The term "solid solution" has been used, and this is a concept essential to an understanding of the science of alloys. Metals in the solid phase exist in the crystalline form, that is, the atoms are located in a regular geometric pattern. Replacement of one of the atoms in a pure metal crystal by the atom of another element, for example, chromium in iron to give stainless steel (Fig. 2), results in a type of "mixture" of atoms known as a solid solution. Again, a stranger atom, such as carbon in iron to give ordinary steel, may simply exist as such in the space within the lattice instead of replacing an iron atom in a given position in a crystal lattice. This is another type of solid solution. In either case, we have a solid solution type alloy.

Suppose now solubility changes with temperature. The temperature is lowered between levels which permit certain atomic mobility. The stranger element or its compound is rejected from solid solution. The resultant alloy is an intimate mixture of small particles of the stranger element or compound surrounded by the crystal lattice of the solid solvent. The small particles of the stranger element may be large enough to see, or they may be submicroscopic. Such alloys are frequently known as precipitation type alloys. The alloy of copper and chromium is a case in point. At some 900° C. an alloy containing 99.5 per cent. copper and 0.5 per cent. chromium exists as a solid solution. As the temperature is lowered the solubility of the chromium in the copper decreases so that at approximately 300° C. less than 0.05 per cent. chromium remains in solid solution, the re-

mainder of the chromium existing as discrete particles, many of which are microscopically visible in the copper crystals.

The eutectic type of alloys results from a sudden change in solubility due to a phase change. This phase change may be from a liquid to a solid or from one solid phase to another. If sufficient solute is present in such alloys the solute is rejected from the solvent as the material undergoes phase

FIG. 2.



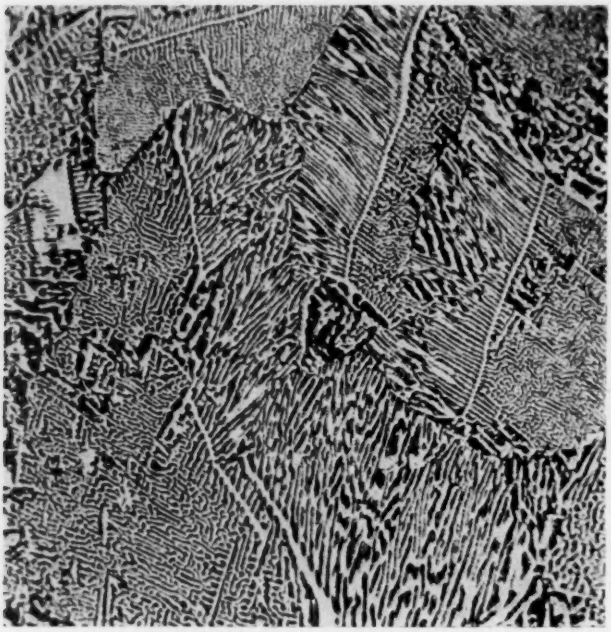
Iron-chromium solid solution and carbides (low carbon chrome iron) etched with aqua regia-glycerine. 250 ×.

changes, generally due to lowering of the temperature. In binary alloys such phase changes generally take place over a very narrow temperature range if not actually at a fixed temperature, and it is common in such cases where time at temperature is sufficient to have the rejected solute present in the form of small plates interleaved with similar plates of the solvent (Fig. 3). When the amount of solute is such that the entire alloy consists of lamellar structure it is known as a

eutectic. If the structure results from a phase change involving two solid phases the structure is similar in character and is known as a eutectoid. We need not concern ourselves with the names given to these structures, but it is important that their nature be visualized.

The alloy of lead and arsenic is a case in point. If we have some 3.3 per cent. arsenic in lead in the liquid phase at

FIG. 3.

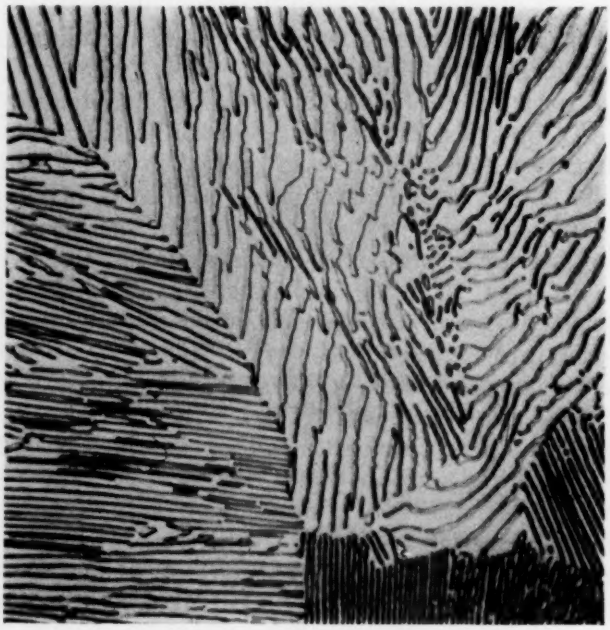


Eutetic alloy. Bismuth-tin-etched with 25 per cent. HCl in alcohol.

300° C. and cool this liquid to 288° C. the liquid phase changes to the solid phase, giving an intimate though macroscopic mixture of lead and arsenic present as interleaved plates or lamellæ. Not only may stranger elements be rejected from solution but such elements may form compounds with the main element and these compounds may be rejected from the solution. In general, such a compound may be considered as a primary constituent similar to an element. Iron-carbon is

a case involving solid phase changes. If an alloy of iron containing approximately 0.9 per cent. carbon is held at 750° C. a complete solid solution exists, the iron being present as the gamma phase or face-centered cubic lattice. As the temperature is lowered to approximately 725° a phase change takes place from gamma to alpha iron, that is, from a face-centered to a body-centered cubic lattice. Solubility of the

FIG. 4.



Pearlite in 0.90 per cent. carbon steel-etched with 5 per cent. picric acid in alcohol. 2000 ×.

carbon in the latter is very much less, so that carbon must be rejected from the solvent, alpha iron. However, the carbon combines with iron to form a compound, Fe_3C, known as cementite, and this forms one part of the lamellar structure typical of a eutectoid, in this case known as pearlite because of its appearance (Fig. 4).

All of the above is representative of the slowly-cooled eutectic or eutectoid type of alloy. If the change in phase is effected rapidly enough so that the rejection cannot take

place as described, intermediate, special structures occur, and correlative with such structures are special properties which may or may not be desirable. This rate of change of atomic arrangement may be influenced not only by the rate of change of temperature but also by the presence of other elements which due to their very presence affect the specific rate of the phase change.

This exposition may seem quite complicated to the layman. However, to one versed in the principles of chemistry and physics it is apparently nothing more than the common teachings of the phase rule, and to the metallurgist it is everyday behavior of his materials.

The purpose of alloying is to produce materials having specific properties or combination of properties. While occasionally we are primarily interested in one property, for example, electrical conductivity or coefficient of expansion, we are frequently interested in more than one property, and the combination of properties that is most important in most alloys involves strength and ductility. Metals may be strengthened by preventing deformation of the grain. Without going into the detail of various theories, we can summarize for our purpose by a few simple statements.

Solid solutions are stronger, but not necessarily much stronger than the pure metals, that is, the presence of a stranger atom as such in the lattice increases the tendency to resist distortion. Much more effective is the presence of stranger particles in the lattice which tend to prevent slip along atom planes. Likewise effective is the matter of grain size, small grains tending to prevent distortion by end blocking of the slip planes. Ductility in solid solution alloys is increased or unaffected; in eutectics it depends on the ductility of the least ductile constituent, and in precipitation type alloys it varies inversely with strength but not necessarily in proportion. As the limiting hardening effect is approached the ductility approaches zero. Thus, there are a number of definite ways in which strong, ductile alloys can be produced. As hardness generally goes with strength, the same is true of the production of hard alloys.

The alloys of the ancients, such as the coinage metals, the tool bronzes, and the Roman brasses, were predominantly of

the solid solution type. This is understandable, in that a simple melting of such alloys together with cooling in any fashion results in a structure having the properties desired. These older alloys were followed by those of the eutectoid type, where reasonably slow cooling resulted in the eutectoid structure. Here again there was great probability of achieving the desired result by simple fusion and normal cooling. In alloys requiring special heat treatments, however, a particular combination of circumstances was necessary to achieve the results, that is, the matching of alloying content and heat treatment. The probability that anyone experimenting in a hit or miss fashion would actually stumble upon the correct combination would appear to be small indeed, but even in 900 B.C. the effect of quenching steel had become apparent, and great progress in the production of new alloys in the period 1850 to 1890 was made in spite of the fact that the principles involved were not understood. This was the period of intensive application of chemical analysis and the tensile testing machine.

At the end of the last century with the advent of the metallurgical microscope, thermal analysis, and convenient hardness testing, the development of new alloys changed from an art to a science which has today reached a still higher level because of concepts made possible by X-ray analysis.

Consider some of the alloys which have been developed in recent years—alloys made possible by an understanding of the principles previously outlined. Duralumin was known but not understood. In the wake of X-ray analysis came Dr. Merica's theory of ultramicroscopic precipitation as a hardness-inducing phenomenon. Shortly thereafter Corson hardened copper with silicide precipitation, Dean hardened lead with precipitated antimony, Mudge hardened nickel-copper by precipitated aluminides, and Masing hardened copper by precipitating beryllides. Thus goes the roll call of modern alloying started by the impact of new understanding.

The copper and silicide story is one of the best examples. Copper dissolves a number of certain elements at high temperatures, and largely retains such elements in solution at low temperatures. By adding more than one copper-soluble

element of such character that the two elements might form a compound which is rejected from solution at low temperature, a precipitation type copper alloy results. Corson found that such elements as nickel, chromium or cobalt on the one hand and silicon or beryllium on the other, each readily soluble in copper at elevated temperatures, form a silicide or beryllide relatively insoluble in copper at low temperatures. Thus by adding proper amounts of these elements to the copper melt and holding the solidified alloy at an elevated temperature, approximately 900° C., a homogeneous solid solution is formed with the added elements well distributed throughout the copper lattice. Quenching retains these elements essentially in solid solution, so that the microstructure is not different from that of pure copper. Reheating at a low temperature, such as 500° C., increases atomic mobility and allows the compound to form with rejection from the solid solution. This results in copper crystals relatively free from atoms of the other elements and particles of the compound dispersed through the crystal. These particles prevent ready deformation of the grain proper, increase the strength and hardness, and retain this condition until the temperature of precipitation is approached. This makes possible the use of copper-base alloys with their naturally high thermal conductivity in such novel applications as combustion motor heads as well as for a wide variety of special applications such as resistance welding electrodes and electric switches.

The aluminum-silicon alloy with or without copper, now so widely used for automobile pistons as well as for a great variety of castings, illustrates the eutectic type of alloy. Here the maximum solubility in the solid phase is low (1.8 per cent.) and the eutectic is formed at a silicon concentration of some 10 per cent. This eutectic has a very peculiar property, in that the net volume change on solidification and cooling to a temperature at which the alloy has acquired strength is very small. Accordingly, molds and patterns are accurately filled and shrinkage stresses practically eliminated. For pistons, much more than 10 per cent. silicon is used in order to have hard silicon crystals to resist wear and lower the coefficient of expansion. The resulting eutectic structure carries with it the strengthening which could be expected

from the juxtaposed plates in each grain. These alloys have the particular property of low density combined with strength. Advantage is taken of low shrinkage in a 5 per cent. aluminum-silicon alloy used for welding aluminum-base articles.

A number of alloys developed to have some singular property are of interest. An alloy of copper, manganese and nickel containing approximately 12 per cent. manganese and 4 per cent. nickel is illustrative. In this alloy constant electrical resistance has been achieved as a function of temperature over the range found in usual living conditions, and in addition the alloy has a low thermoelectric force when coupled with copper. It is used in electrical measuring instruments similar to the meter whose reading determines your monthly electric light bill. Again, an alloy of nickel and iron which contains some 37 per cent. nickel possesses a coefficient of expansion that approaches zero over a limited temperature range, including the one in which we live, and is therefore used for surveyors' tapes, pendulum stems and thermostatic devices. The phenomenon is probably due to the same mechanism as that of the copper-manganese alloy previously mentioned, namely, a perfectly reversible formation and dissociation of intermetallic compounds within the solid solution—what the modern physicist would call "order disorder" variant with temperature. Iso-elastic is another case in point. This is an iron-base alloy with 37 per cent. nickel, 7.5 per cent. chromium and 0.5 per cent. molybdenum having a practically constant modulus, as a function of temperature—again over a limited but desirable range and likewise exhibiting extremely low plastic deformation and mechanical hysteresis. It was developed by deForest for accurate weighing mechanisms similar to those found in the modern butcher shop. Similarly, alloys of iron, chromium and nickel with manganese, vanadium, silicon or aluminum highly complex in character have been developed. These have coefficients of expansion approximating that of glass and are used for leads into glass bulbs containing electrical apparatus. The alloys having very great hardness are frequently based on a large proportion of hard particles of compound held in a small proportion of solid solution. Stellite, essentially compounds of tungsten, chromium and carbon in a

precipitation type cobalt base alloy, and the more recent tungsten-carbon-cobalt, essentially tungsten carbide in a cobalt-base solid solution, are typical. They are used for cutting tools and dies, having the property of retaining hardness at elevated temperatures.

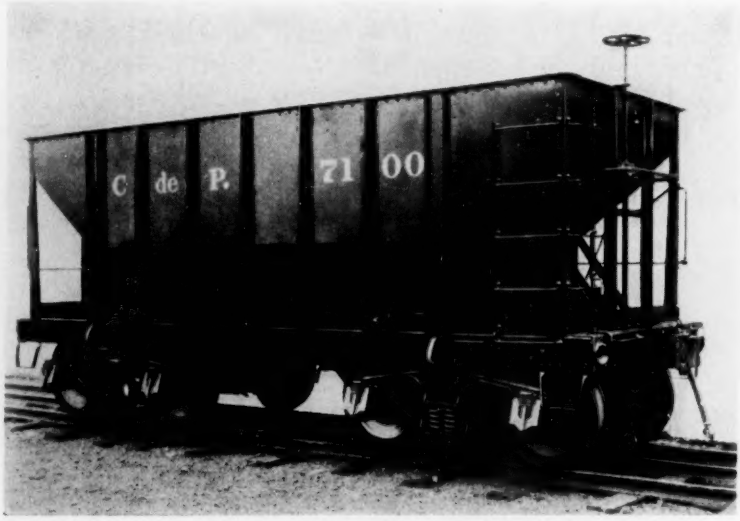
A whole series of alloys, containing essentially 50 to 80 per cent. nickel, remainder iron, has been developed for maximum magnetic permeability with low hysteresis and remanence, and are used in long distance telephone and in cable lines. Another series developed for maximum remanence and hysteresis comprises iron, nickel, aluminum and cobalt, illustrated by the powerful permanent magnets used in radio speakers. In this field the metallurgist is still largely groping, having no comprehensive theory to explain the mechanism or to guide his work. These alloys are illustrative of another point, namely, that nature is often perverse. These permanent magnet alloys have so little ductility either at room or elevated temperature that they can be produced in usable form only by direct casting or by powder compacting.

Probably the most fascinating of all of the alloy systems are those based on iron as this is the only common inexpensive element which undergoes a solid phase change. The nature of this phase change in the simple iron-carbon alloy is fairly well understood, and has already been described to you by Dr. Johnston in his talk last year. Briefly, this phase change results in various types of mixtures of ferrite and iron carbide due to high solubility of carbon in the high temperature γ phase and low solubility in the low temperature phase, α. The particular type of mixture may be controlled by affecting the rates of phase change either by altering the rate of cooling of the article or by altering the specific rate of phase change for a given rate of cooling by means of alloy additions. The modern low alloy, high strength steels are a case in point. Such steels, in view of the fact that they are used as structural members of considerable magnitude, are not subject to heat treatment, but are used as cooled in the air from the rolling operation, or after a simple reheating and air cooling. Cranes, buckets and railroad cars (Fig. 5) are typical of structures built of these steels. The alloy addition is so adjusted with respect to carbon that a fine pearlitic structure results from

this type of cooling, which structure correlates with strength and ductility. But even greater demands have been made on such alloys. They not only must have increased strength and ductility, but also must retain this strength and ductility after specific heating operations such as occur in welding, and further, due to the fact that they are used in lighter sections, must show increased corrosion resistance.

There are many types of alloy steels intended for this purpose and all are based on the same general principles.

FIG. 5.



They all follow the same fundamental rules, namely, they must contain an element which primarily affects the transformation, such as chromium, manganese or nickel; an element to give corrosion resistance, such as phosphorus or copper, or both; and another element to control yield point or modify the transformation with different cooling rates, such as silicon, vanadium or molybdenum.

Consider one of the better known of the modern high strength structural steels. To meet the welding requirements carbon must be kept low. In general, it has been determined that in the structural alloy steels of this type no more than

0.14 per cent. carbon is permissible. To get the type of fine pearlite required with carbon at such a low level, chromium, manganese or nickel may be added. In view of the fact that corrosion resistance is desired along with the other properties, phosphorus and copper are used. Phosphorus tends to have an effect similar to carbon in producing fine pearlite. There will be a tendency for the pearlite to be too fine, which means reduced ductility, and accordingly the carbon is held at a still lower level, namely, 0.10 per cent. maximum. The copper tends to promote surface checking on hot working, so that this element is restricted to 0.40 per cent. In order to obtain the required corrosion resistance with the copper at this level phosphorus is increased to some value above 0.10 per cent., and to eliminate any embrittling effect of this phosphorus, chromium is chosen as the strengthening element and is placed at some value above 1 per cent. Manganese would tend to affect the transformation so as to get too fine a pearlite, so this is kept to a minimum, at approximately 0.30 per cent. In order to have a relatively high yield ratio, that is, to have strengthening by solid solution affecting the ferrite proper, silicon is carried at a relatively high level, above 0.5 per cent. All of this results in an alloy which at first glance appears to be tremendously complicated. It will be noted that this steel contains maximum 0.10 per cent. carbon, approximately 1.10 per cent. chromium, 0.30 per cent. manganese, 0.11 per cent. phosphorus, 0.40 per cent. copper and 0.6 per cent. silicon. It will also be noted that there is a very definite reason for each and every one of these alloying elements and for their proportions, that is, the metallurgist started out to get a combination of properties and by using the elements as building blocks and noting and balancing the effect of each he has been able to synthesize a steel having all the required properties.

Consider another of the low alloy steels. Carbon has been retained at 0.14 per cent. maximum to get the required fine pearlite without too much hardening when cooled from a very high temperature. Chromium at approximately 0.5 per cent. and manganese at approximately 0.8 per cent. work together to affect the transformation. Silicon is used at 0.7 per cent. to increased yield point, and copper at 0.4 per

cent. to increase corrosion resistance. In addition, zirconium is added to improve ductility of the ferrite proper, again attention being paid to balancing the various alloying components. And still another case, with carbon under 0.14 per cent., copper is placed at 1 per cent. or more for corrosion resistance, nickel is chosen at 1 per cent. or more as the strengthening element because it eliminates copper surface checking and if the pearlite is not sufficiently fine, depending on the amount of nickel, some 0.2 per cent. molybdenum may be further added—again the building and balancing process.

In the engineering steels used for automotive and machine tool construction the size and shape of the article are such that heat treatment is practicable, thus differing from the structural steels. A variety of alloy steels is available, each with some particular property. The plain chromium steels tend to form fine carbides which give wear resistance. These are found in ball bearings. Likewise in lower carbon ranges the fine pearlite induced by chromium promotes strength and ductility. The nickel and nickel-chromium steels are readily carburized and are also found in parts where strength and ductility are of paramount importance. Gears are typical. Vanadium is used to augment the carbide-forming effect and reduce grain size. Molybdenum affects the transformation so as to give fine pearlite and is generally used in combination with other elements.

All of the foregoing refers to the problem of physical properties of alloys, but chemical properties, or more pertinently resistance to corroding media, are likewise of great interest and practical importance. In general, there are two methods of obtaining corrosion resistance. One is to use the so-called noble metals, or the highly passive element tantalum. Alloys of gold and silver or platinum and rhodium are illustrative. In such alloys there is no chemical attack by a large variety of chemical media. The second method is to so proportion and select the alloys that the products of corrosion act to protect the alloy proper from the corroding medium. This mechanism is based on which is commonly known as the film theory. Chromium, for example, oxidizes readily, but the oxide film is tenacious and continuous so that attack may cease almost at once. This phenomenon is the basis

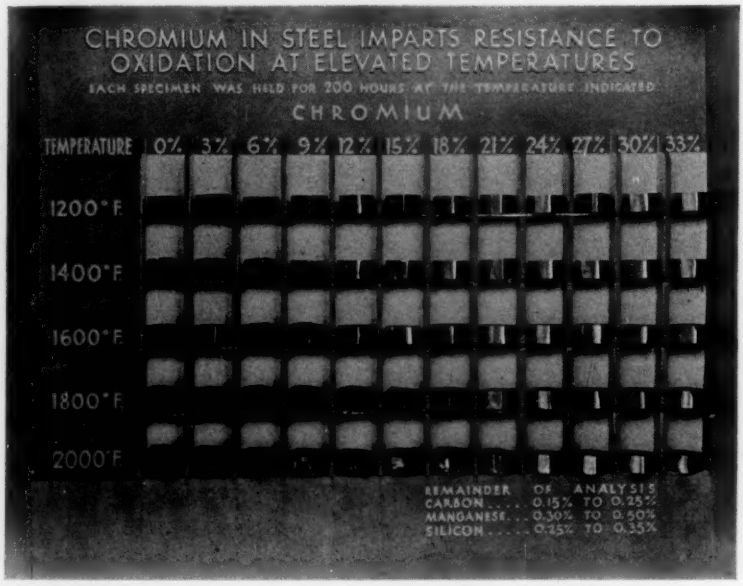
for a most interesting series of alloys, commonly known as stainless steels.

The first stainless steel of the cutlery type was produced by Brearley, and contained some 14 per cent. chromium and 0.30 per cent. carbon. He found that corrosion resistance to oxidizing media was excellent when the material was in the heat treated condition, that is, when the state of heat treatment was such that the chromium was not tied up to the carbon in the form of carbide to such a degree that it could not act to form a continuous film. Cutlery is illustrative. Shortly thereafter it was found that lowering the carbon gave an alloy of the same degree of corrosion resistance without necessitating heat treatment, and further that increasing the chromium content increased the resistance to corrosion, particularly in oxidizing media. Such steels are found in hub caps, nitric acid handling equipment and many other decorative and chemical applications. Closer study of the phenomenon has led to the belief that the film is not entirely passive in nature but is dissolved slowly as a function to time, and that the rate of corrosion in a given medium is a function of the rate of removal and rate of reformation of the film and underlying alloy. Thus, the finding that the higher the chromium the greater the corrosion resistance is logical, as the increased chromium would undoubtedly increase the rate of film formation.

It was further found that in the higher chromium alloys not only was there a marked resistance to corrosion but also a marked resistance to atmospheric oxidation at elevated temperatures (Fig. 6). This had been previously noted in chromium-nickel alloys. This resistance to oxidation seems to follow the same general mechanism as resistance to corrosion, that is, the material oxidizes very rapidly at first and the products of oxidation protect the alloy proper, effectively stopping progressive oxidation. Furnace parts are illustrative. As the chromium is increased in low carbon iron alloys the well-known alpha-gamma transformation occurs over an ever more narrow temperature range, so that in the carbon-free alloys with over 12 per cent. chromium the transformation is completely suppressed, and with moderately small amounts of carbon the same is true at slightly higher chromium con-

tents. Thus, one of the most valuable properties of the iron alloys is lost, namely, the ability to refine the grain without mechanical work. This means that once such an article has been wrought to its final shape every high temperature heating operation serves to increase the grain size with correlative loss of ductility. Inability to obtain grains in castings finer than the initial as-cast grain size, which is comparatively large, has seriously hampered the industrial use of these

FIG. 6.

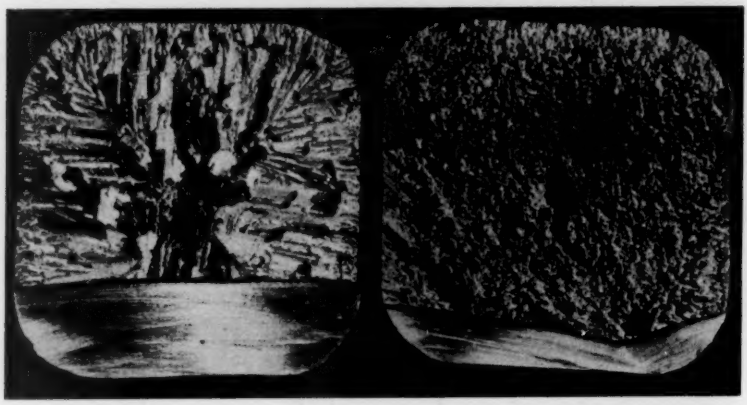

CHROMIUM IN STEEL IMPARTS RESISTANCE TO OXIDATION AT ELEVATED TEMPERATURES
EACH SPECIMEN WAS HELD FOR 200 HOURS AT THE TEMPERATURE INDICATED
CHROMIUM
TEMPERATURE
0% 3% 6% 9% 12% 15% 18% 21% 24% 27% 30% 33%
1200°F.
1400°F.
1600°F.
1800°F.
2000°F.
REMAINDER OF ANALYSIS
CARBON . . . 0.15% TO 0.25%
MANGANESE . . . 0.30% TO 0.50%
SILICON . . . . . 0.25% TO 0.35%

materials, and it is only more recently that this problem has been solved to a large degree by the addition of nitrogen. This results in material of very much finer grain (Fig. 7) and lesser tendency toward grain growth in a given heating cycle. The reasons for this nitrogen addition and its effect will be discussed at greater length together with master alloys.

One other attack on this problem has been the addition of further alloying elements which tend to restore the natural tendency of the iron-carbon base alloy to transform. Just as chromium promotes alpha iron, other elements such as

manganese, nickel, copper and nitrogen, not to mention carbon, tend to promote gamma iron. Thus a balanced proportion of such elements together with the chromium results in high chromium steel which undergo transformation and are accordingly amenable to heat treatment for grain refinement or other purposes. By adding still larger amounts of austenite-forming elements such as nickel or manganese it is possible to produce a high chromium alloy which is completely austenitic at some elevated temperature and which undergoes the transformation so sluggishly that it is possible to suppress

FIG. 7.



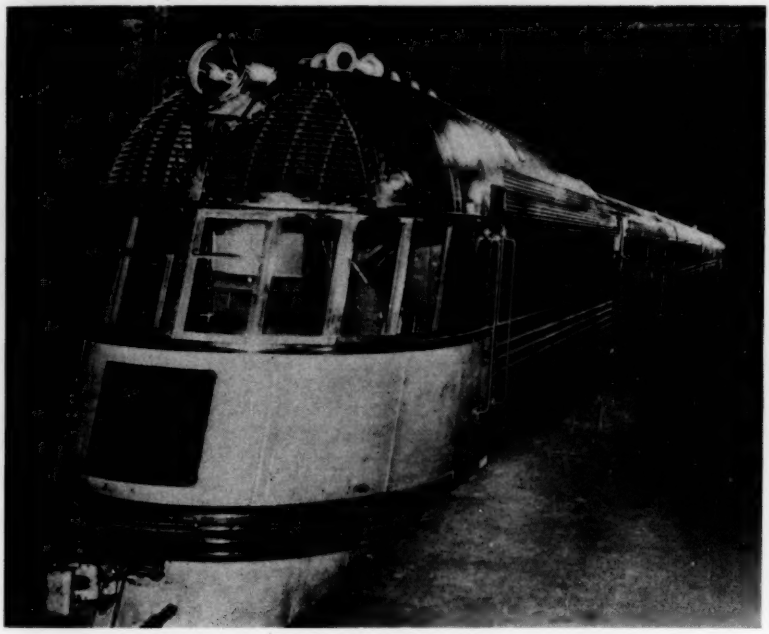
28 per cent chromium steel as cast, without and with a nitrogen addition.

it by rapid cooling. The austenitic structure possesses much greater ductility than the ferritic structure and facilitates such special operations as deep-stamping and cold forming.

The best known alloys in this class contain some 18 per cent. chromium and 8 per cent. nickel with low carbon. Alloys of this type are soft and highly malleable in the rapidly cooled state. They form the basis for equipment in the dairy and food industries, decorative trim and the chemical industry in general. Similar to many non-ferrous alloys, they may be cold worked to produce high strength without too great a loss of ductility. The "18-8" alloy in this condition is the essence of the modern light weight, high strength construction

found in the recent streamlined transportation structures such as railroad trains (Fig. 8), motor trucks and airplanes. These alloys contain comparatively small amounts of carbon, 0.07 per cent. maximum being a common specification. However, even this small amount of carbon may cause undesirable effects. Increasing the temperature to a point where there is sufficient atomic mobility so as to allow the structure

FIG. 8.



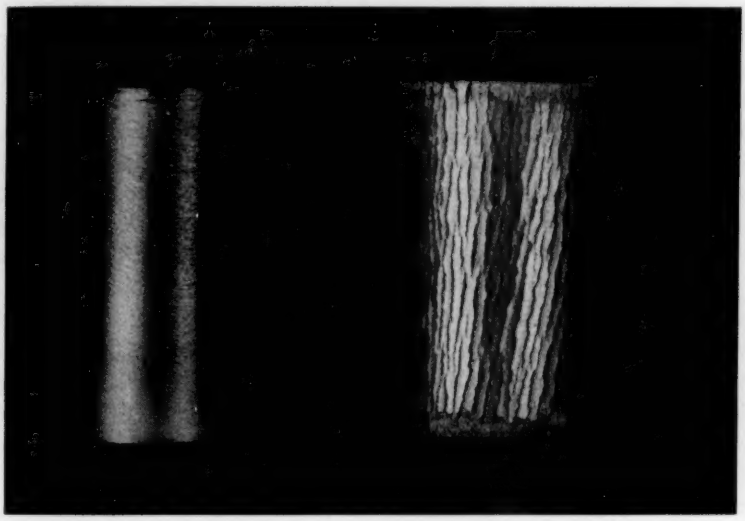
to change toward the equilibrium condition results in transformation of austenite to ferrite. Such transformation first takes place at the point of maximum internal stress, namely, the grain boundaries. Carbon being much less soluble in ferrite than in austenite is precipitated at these grain boundaries in the form of carbide. When these carbides are present in the grain boundaries the material exhibits lower ductility and corrosion resistance. In most applications of the standard 18-8 low carbon material the only exposure to heat

at the temperature implied above happens in welding, and this is for such a short time that the degree of the precipitation is slight. Thus, for many engineering purposes the loss of ductility and corrosion resistance is of such small magnitude as to be inconsequential. In fact, in the case of the high speed resistance welding used extensively within walking distance of this Institute the time at temperature is so short that the intergranular phenomenon does not take place at all. However, where corroding conditions are severe, where relatively thick material involves a long time in the welding operation, or finally where the material is used at the temperature of precipitation the loss of ductility and corrosion resistance may be serious. The precipitation may be eliminated by reducing the carbon to practically zero. For this purpose it is necessary to maintain a carbon content under 0.01 per cent., and this is not now commercially feasible. Again the science of alloys comes to the rescue, it having been found that a suitable amount of a very strongly carbide-forming element, such as columbium or titanium, will form carbides and practically eliminate carbon from solid solution in the austenite. Thus, there is no carbide precipitation on heating to the precipitation temperature and no correlative loss in corrosion resistance. In other words, the phenomenon of intergranular corrosion disappears in these steels when columbium is present to the extent of ten times the carbon (Fig. 9), and lesser amounts may be practically effective. This is a boon to the chemical industries where many of the recently developed reactions are carried out in stainless welded equipment.

Again, under certain corroding conditions where the media are non-oxidizing in nature corrosion of the iron-chromium alloys may take place at an undesirable rate. The addition of nickel to the extent above mentioned, namely 8 per cent. or thereabouts, renders the film and underlying material much more resistant to such media so that the net corrosion resistance may be markedly increased. This may be correlated with the corrosion resistance of metallic nickel. In some media, particularly those containing halogen radicals, molybdenum has a similar major effect on corrosion resistance of these alloys. The use of 2 or 3 per cent. molybdenum to

improve corrosion resistance and the use of columbium to inhibit intergranular corrosion would seem to be mutually compatible, and indeed such steels have been made. However, both molybdenum and columbium are alpha-forming elements, with the result that when both of these elements are present in addition to the chromium it is difficult to retain the austenitic condition to a satisfactory degree. Again the science of alloying gives the answer, in that an increase in the

FIG. 9.



Effect of columbium on intergranular corrosion in 18-8 steel. Both samples heated at 650° C. four hours and air-cooled then immersed in boiling acidified copper sulfate solution for one and one-half hours and then bent. Sample on left was columbium-bearing, the one on the right a plain 18-8 steel.

austenite-forming elements, such as manganese as well as nickel, counterbalances this effect of the molybdenum and columbium and results in steels having the desired degree of austenite stability and correlative properties which render it suitable for use in a wider range of chemical equipment, and particularly improves resistance to sea water and brines.

The subject of master alloys mentioned in the earlier part of this discussion has been purposely left to the last as these alloys are primarily of interest to the manufacturing metal-

lurgist. The purpose of these alloys is to enable ready introduction of alloying elements whether for the direct purpose of alloying in line with objectives already discussed, for the purpose of eliminating undesirable matter, or for creating specific conditions leading to desired types of crystallization on cooling. The master alloys are industrially known either as hardeners or as ferro-alloys, depending on their composition. Typical of the hardeners are alloys of 50 per cent. silicon-aluminum, 30 per cent. silicon-copper, 30 per cent. manganese-copper, 10 per cent. chromium-copper, 15 per cent. phosphorus-copper and 10 per cent. nickel-aluminum. Typical of the ferro-alloys are 50 and 75 per cent. ferrosilicon, 70 per cent. ferrochromium and 80 per cent. ferromanganese, each with high or low carbon, and 40 per cent. ferrovanadium. In the melting of any alloy the problem of deleterious effects produced by unavoidable stranger elements is ever present. Oxygen from the air or from refining slags is most prevalent. Nitrogen from the air, sulphur from the ore or from reducing media, and phosphorus from the ore are typical.

In the manufacture of non-ferrous alloys deoxidation is generally unnecessary, copper and nickel being important exceptions. The addition of a variety of agents, the most common being phosphorus, silicon, calcium, and zirconium, is used in these cases. These combine with the oxygen in the melt to form oxides, which are either eliminated or harmless. Excess quantities of deoxidizers may, however, enter into a liquid solution and form either a solid solution or eutectic alloy. When this happens the material falls in the class of alloys previously described, and counterbalancing alloys and additions may be necessary, or changing the deoxidizer may result in elimination of the new problem. For instance, iron and chromium are reported to offset the small amounts of phosphorus which might otherwise be in solid solution in copper deoxidized with this element, and chromium may be used to offset similar effects of silicon.

The problem of deoxidizing non-ferrous alloys has been relatively simple. However, in the ferrous alloys, particularly the steels, the problem is much more complex and here the term deoxidation is so broad as to include reaction with other elements as well. For instance, oxides and sulphides

may be retained as a network in the initial grain boundaries of a casting and may seriously affect the ductility. In an ingot to be rolled the inclusions may be present as alumina, silicates or complex sulphides so that they roll to stringers in the steel and result in local weakness, initiating fatigue failure.

The science of deoxidizing steels so as to avoid undesirable types of inclusions is of comparatively recent origin. Herty has shown that in the initial deoxidation of a steel bath the use of a special alloy of silicon and manganese results in inclusions of just sufficient solubility and viscosity at usual steel bath temperatures so that they readily agglomerate and float out of the bath. The addition of zirconium to eliminate nitrogen from an active part in the final alloy by formation of zirconium nitride is another finding of more recent years. The use of manganese to eliminate iron sulphide by substitution of iron and formation of manganese sulphide is old in the art. It was long thought that steel could not be made without a manganese addition due to the ever-present sulphur. However, Becket has shown that zirconium may replace manganese in this respect, the mechanism being essentially the same.

After the steel is taken from the furnace a final deoxidation in the ladle is necessary. Silicon and aluminum are widely used for this purpose. This brings with it a certain danger of inclusion formation, but as a rule the oxygen content is sufficiently low so that this matter is not serious. However, when large additions of aluminum are used in order to obtain inherently fine-grained steels the inclusion problem may again become serious. This matter of fine-grained steels is illustrative of more recent developments. It has been found that if a large aluminum addition is made to the steel the tendency for grain growth in the austenitic phase is markedly reduced, the steels so treated being known as inherently fine-grained steels. Vanadium has been used similarly for many years, and without the correlative inclusion problem, but the cost of this element has discouraged its use in the less important alloy steels.

The mechanism of production of fine-grained steels has not been well established as yet. One theory holds that the

small grains are produced by the presence of small oxide or carbide particles acting as nuclei which give seeding action similar to that of dust in precipitation from saturated liquid solutions. Another holds that it is due to the actual presence of small amounts of the elements in question in solid solution. Regardless of the theory, we are faced with the need for an addition agent which will produce the fine grain, but at the same time not present an inclusion problem or an economic problem. As a result of a study of inclusion types, complex alloys have been developed. One of the more recent and effective of these contains silicon, aluminum, vanadium and zirconium in balanced proportions. This gives freedom from undesirable types of inclusions, and the combined effect of three grain-refining elements—aluminum, vanadium and zirconium—accomplishes the major objective by producing inherently fine-grained steel.

Master alloys, particularly those in the ferro-alloy category, may also be used for special purposes. It was earlier mentioned that nitrogen in high chromium steels resulted in a relatively fine grain in the cast state. This is effectively accomplished by adding the chromium in the form of a master alloy of iron, chromium and nitrogen, commonly known as high nitrogen ferrochromium. Whether because of the seeding action of the chromium nitride or for some other reason not explained the grain size of the cast structure is markedly reduced and tendency to grain growth on long exposure at elevated temperatures in service is likewise reduced. This also has a correlative effect on the hot working properties of ingots and billets, the fine-grained structure in this case being appreciably more ductile, particularly at elevated temperatures. As a result, the nitrogen addition is almost universally used today in high chromium irons.

Cast iron is a most complex alloy. In view of the fact that very little was demanded of cast iron in the past the real complexity of its nature was not appreciated. Only with the advent of high strength cast irons and short cycle malleable cast irons has the true nature of cast iron been investigated. There are two distinct problems in cast iron which are unfortunately interrelated. One is to produce a cast iron in which the graphite is suitably distributed, generally as rela-

tively fine graphite flakes. The other is to affect the matrix, essentially an iron carbon alloy, by means of alloying additions in a manner entirely analogous to the way in which alloys affect steel proper. It is not surprising that most of the alloying additions, such as chromium, manganese and molybdenum, when used in proper proportions tend to produce a pearlitic matrix. However, most of these alloying additions likewise tend to form carbide rather than graphite, so that other alloying additions are necessary to balance this effect and prevent the alloy from forming a cementite mass rather than a gray cast iron. The best known graphitizers are silicon, nickel and copper, although calcium, aluminum, and zirconium are also of interest. Metallurgists have developed a number of master alloys, combining elements from each group. Many of these are ferro-alloys. They are already industrially successful and with the increased understanding of the exact effect of each element, alone or in combination, even more effective addition alloys are to be expected.

From all of the above we have seen that alloying has progressed from an art to a science, and while we have learned much, particularly in the last twenty years, there is still a great deal to be learned. The broad subject of solid solution and intermetallic compound formation, together with the previously mentioned concept of "order disorder," is still being intensively studied by the physicists, and as yet can hardly be said to have been applied in the science of alloys. Certainly within the next decade this concept will be reduced to the category of a tool for the metallurgist. Again, the structure of the atom proper is under constant investigation by the physicist, and a better understanding of the significance of the nature of electron shells will do much toward answering the problem of why certain elements are soluble in each other in the solid phase under certain conditions and form intermetallic compounds under other conditions. Further, while we have long realized the analogy between the reactions and phase changes of metallurgy and those that are normally considered in the field of chemistry, the effect of diffusion velocities on the specific rate of such metallurgical reactions is only now being given serious consideration. The quantitative correlation of rates of diffusion and rates of transformation

will do much to increase the ability of the metallurgist to synthetically produce and control their behavior. Thus, we may look forward to a future of alloys tailor-made for specific purposes. Such alloys will result in much more efficient engineering and overall economy.

It has often been said that man has passed from the Stone Age, through the Bronze Age, through the Iron Age and is at present in the Steel Age. If we thus take man's more common materials to designate the Age in which he lives it is certainly true that we are just entering the Age of Alloys and that man's technological future is indeed bright.

BEAT THEORY OF NON-LINEAR CIRCUITS.

BY

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ABSTRACT.

The recent improvement in transmission line regulation by the introduction of series capacitance has stimulated an already great industrial interest in non-linear circuits. By proper choice of circuit elements and saturation curve in the non-linear circuit of Fig. 1 it seems possible to obtain a current wave of any shape whatsoever, limited only by the fact that the graph of the current be single-valued and possess a continuous first derivative. In this paper a beat theory of non-linear circuits is developed by which the curious wave forms (as examples, see Figs. 4–8) are explained and accurately computed. The complete solution of the non-linear differential equation is obtained and the theory has been checked by means of numerical integration performed on the differential analyzer. A method of obtaining a linear circuit which is the equivalent of the non-linear circuit is given. A criterion is derived by which beats in such circuits can be eliminated.

I. EXPERIMENTAL PHENOMENA.

It is well known that when the voltage $E \sin t$ is applied to the circuit of Fig. 1, E being in excess of the resonant voltage and $R_2 = \infty$ then the current response is a periodic and sharp peaked wave. A criterion for the resonant voltage as a function of the circuit constants and constants of the saturation curve has been given in a previous paper.[2]

When the above voltage is applied, E being less than the resonant voltage, but yet large enough to render the circuit non-linear, then for certain industrial values of the circuit parameters the following curious results are obtained:

(*a*) The steady-state current response may not even be periodic or if so then of very long period. The differential analyzer solution, shown in Fig. 4 curve (*a*), is a steady state solution for flux linkages and illustrates this kind of response.

(*b*) Unlike a linear circuit, the nature of the *steady-state*

[2] See numbered references in the bibliography.

current wave is dependent upon the phase of voltage at which the circuit is closed.

(*c*) Both the differential analyzer solution and oscillograms indicate that the steady-state solution may possess a certain type of instability. This is illustrated in Fig. 7. In this case the amplitude of the current (flux) increases at fixed intervals.

(*d*) This unstable condition is sometimes *increased* by *decreasing* the voltage. This is illustrated in Fig. 4.

(*e*) These undesirable phenomena can be eliminated and a sinusoidal current response obtained if the value of R_2 is properly chosen relative to the other circuit constants and the constants of the saturation curve.

A satisfactory theory of this circuit should not only explain qualitatively the phenomena here displayed but should give analytical methods of computing the electrical and magnetic response of the circuit. And what is of industrial importance is to give a method of obtaining the most economical value of R_2 for which a given circuit will yield sinusoidal steady-state response.

II. MATHEMATICAL THEORY.

Differential Equations. The differential equations for the currents in the circuit of Fig. 1 are

$$\frac{d\psi}{d\theta} + R_1 i_1 + L\frac{di_1}{dt} + \frac{1}{C}\int (i_1 - i_2)dt = E\sin(\theta + \theta_0), \quad (1)$$

$$i_2 R_2 = \bar{x}_c \int (i_1 - i_2)d\theta, \quad (2)$$

where R_2, R_1, $\frac{1}{C} = \bar{x}_c$, L, ψ, i_1, and i_2 are per-unit quantities, i.e., the first differential equation has been divided by the coefficient of $\frac{d\psi}{d\theta}$ after the frequency of the applied voltage has been reduced to $1/2\pi$ by change of independent variable. The equation of the magnetization curve is taken to be

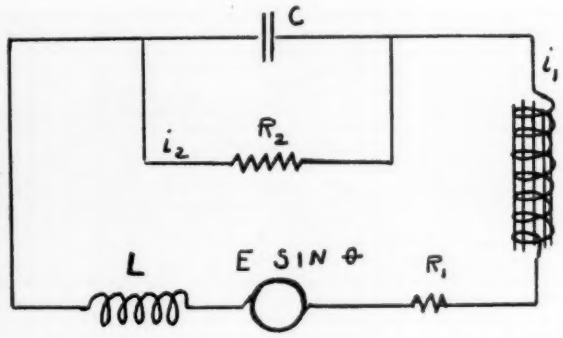
$$H = \alpha i = x - a_3 x^3 + a_5 x^5, \quad (3)$$

where $x = \frac{B}{B_0}$ and B denotes total flux. (See Fig. 3.) The

meaning of B_0, α, a_3, and a_5 is given in Part IV on Experimental Checks. Combining Eqs. (1, 2, 3) we have

$$\alpha B_0 \frac{d^2x}{d\theta^2} + \left[\frac{\bar{x}_c B_0 \alpha}{R_2} + R_1(1 - 3a_3x^2 + 5a_5x^4) \right] \frac{dx}{d\theta} + \frac{\bar{x}_c(R_1 + R_2)}{R_2}(x - a_3x^3 + a_5x^5) = \frac{\alpha\sqrt{R_2^2 + \bar{x}_c^2}}{R_2} E \cos\left(\theta + \theta_0 - \tan^{-1}\frac{\bar{x}_c}{R_2} \right),$$

FIG. 1.



The last equation can be written

$$M\frac{dx}{d\theta} + R(x + F'm^2x^3 + G'm^4x^5) + x_c \int (x + Fm^2x^3 + Gm^4x^5)d\theta = \frac{\alpha\sqrt{R^2 + \bar{x}_c^2}}{R_2} E \sin(\theta + \theta_0 - \phi_0), \quad (4)$$

where $R = \frac{\bar{x}_c B_0 \alpha}{R_2} + R_1$, $x_c = \frac{\bar{x}_c(R_1 + R_2)}{R_2}$, $\phi_0 = \tan^{-1}\frac{\bar{x}_c}{R_2}$, $M = \alpha B_0$, $F' = -\frac{R_1}{R}a_3$, $G' = \frac{R_1}{R}a_5$, $F = -a_3$, and $G = a_5$, and m^2 is a parameter whose significance is given in Part IV.

A Steady-State Solution. There exists a formal solution of Eq. (4) of the form

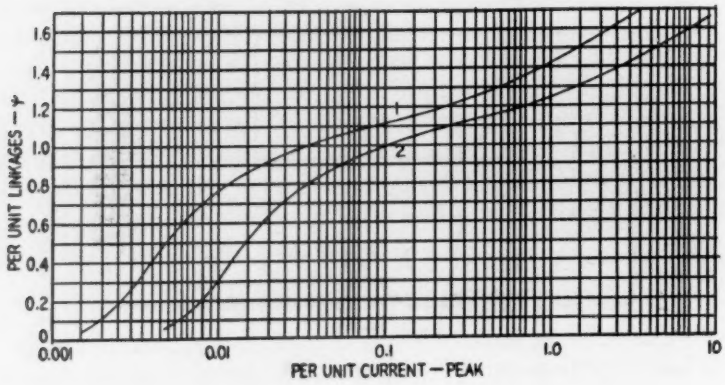
$$x = x_0 + x_2m^2 + x_4m^4 + x_6m^6, \quad (5)$$

which converges for m^2 sufficiently small or for $m^2 = 1$ provided F', G', F, and G are sufficiently small. On substituting Eq. (5) in Eq. (4) and equating like powers of m^2 we obtain the equations

$$M\frac{dx_0}{d\theta} + Rx_0 + x_c\int x_0 d\theta = \bar{E}\alpha \sin(\theta + \theta_0 - \phi_0), \tag{6}$$

$$M\frac{dx_2}{d\theta} + Rx_2 + x_c\int x_2 d\theta = -RF'x_0^3 - x_cF\int x_0^3 d\theta, \tag{7}$$

FIG. 2.



$$M\frac{dx_4}{d\theta} + Rx_4 + x_c\int x_4 d\theta = -R(3F'x_0^2x_2 + G'x_0^5) - x_c\int (3Fx_0^2x_2 + Gx_0^5)d\theta, \tag{8}$$

$$M\frac{dx_6}{d\theta} + Rx_6 + x_c\int x_6 d\theta = -R[3F'(x_0x_2^2 + x_0^2x_4) + 5G'x_0^4x_2 + H_1'x_0^7] - x_c\int [3F(x_0x_2^2 + x_0^2x_4) + 5Gx_0^4x_2 + H_1x_0^7]d\theta. \tag{9}$$

The general solution of Eq. (6) is

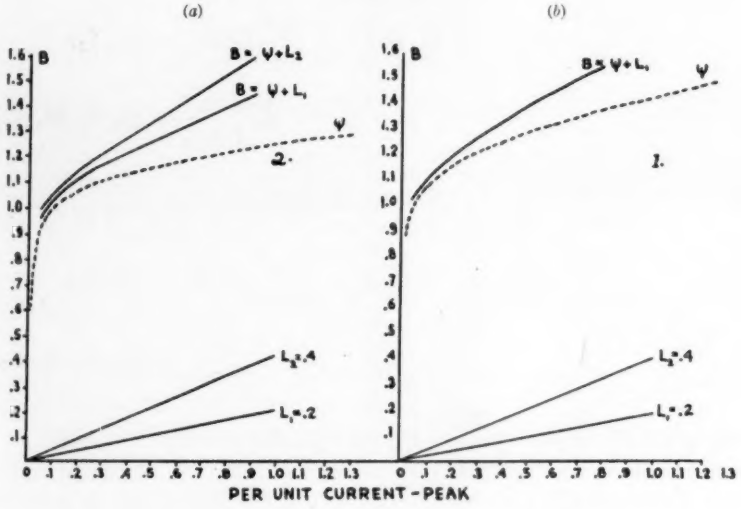
$$\bar{x}_0 = C_1e^{-m_1\theta} + C_2e^{m_2\theta} + V\sin\xi,$$

where C_1 and C_2 are arbitrary constants, z_0 is the impedance,

$V = \dfrac{\bar{E}\alpha}{z_0}$, and $\xi = \theta + \theta_0 - \phi_0 - \varphi$. At this point we are interested in only *a* steady-state solution of Eq. (6). If $\bar{x}_0$ is substituted in Eq. (7) all terms containing $e^{-m_1\theta}$ or $e^{-m_2\theta}$ will vanish in the final solution for large θ. Consequently, substituting $x_0 = V \sin(\theta + \theta_0 - \phi_0 - \varphi) = V \sin \xi$ in Eq. (7) and obtaining the steady-state solution we have

$$x_2 = A_2V^3 \sin(\xi - \phi) + B_2V^3 \cos(\xi - \phi) + \bar{A}_2V^3 \sin(3\xi - \phi_3) + \bar{B}_2V^3 \cos(3\xi - \phi_3),$$

FIG. 3.



where

$$A_2 = \frac{-3}{4z_0}RF', \qquad B_2 = \frac{3x_cF}{4z_0}, \qquad \bar{A}_2 = \frac{RF'}{4z_3},$$

$$\bar{B}_2 = \frac{-x_cF}{12z_3}, \qquad z_0 = \sqrt{(M - x_c)^2 + R^2},$$

$$\phi_0 = \tan^{-1}\frac{M - x_c}{R}, \qquad z_3 = \sqrt{\left(3M - \frac{x_c}{3}\right)^2 + R^2},$$

$$\phi = \tan^{-1}\frac{M - x_c}{R}, \qquad \phi_3 = \tan^{-1}\frac{3M - x_c/3}{R}.$$

The substitution of these values for x_0 and x_2 in Eq. (8) yields eventually after combining terms

$$x_4 = D_0 V^5 \sin \xi + A_4 V^5 \sin (\xi - \phi) + B_4 V^5 \cos (\xi - \phi) + \bar{A}_4 V^5 \sin (3\xi - \phi_3) + \bar{B}_4 V^5 \cos (3\xi - \phi_3),$$

where

$$D_0 = \frac{9}{16} \frac{(R^2 F'^2 + x_c^2 F^2)}{z_0^2},$$

$$A_4 = -\frac{5}{8}\frac{RG'}{z_0} - \frac{3}{2}\frac{RF'}{z_0^2}(RA_2 + M_1 B_2) + \frac{3}{4}\frac{RF'}{z_0 z_3}(R\bar{A}_2 + M_3 \bar{B}_2) - \frac{3}{2}\frac{x_c F}{z_0^2}(RB_2 - M_1 A_2) + \frac{3}{4}\frac{x_c F}{z_0 z_3}(R\bar{B}_2 - M_3 \bar{A}_2),$$

$$B_4 = +\frac{5}{8}\frac{x_c G}{z_0} - \frac{3}{2}\frac{RF'}{z_0^2}(RB_2 - M_1 A_2) + \frac{3}{4}\frac{RF'}{z_0 z_3}(R\bar{B}_2 - M_3 \bar{A}_2) + \frac{3}{2}\frac{x_c F}{z_0^2}(RA_2 + M_1 B_2) - \frac{3}{4}\frac{x_c F}{z_0 z_3}(R\bar{A}_2 + M_3 \bar{B}_2),$$

$$\bar{A}_4 = +\frac{5}{16}\frac{RG'}{z_3} - \frac{3}{2}\frac{RF'}{z_3^2}(R\bar{A}_2 + M_3 \bar{B}_2) + \frac{3}{4}\frac{RF'}{z_0 z_3}(RA_2 + M_1 B_2) - \frac{1}{2}\frac{x_c F}{z_3^2}(R\bar{B}_2 - M_3 \bar{A}_2) + \frac{1}{4}\frac{x_c F}{z_0 z_3}(RB_2 - M_1 A_2),$$

$$\bar{B}_4 = -\frac{5}{48}\frac{x_c G}{z_3} - \frac{3}{2}\frac{RF'}{z_3^2}(R\bar{B}_2 - M_3 \bar{A}_2) + \frac{3}{4}\frac{RF'}{z_0 z_3}(RB_2 - M_1 A_2) + \frac{1}{2}\frac{x_c F}{z_3^2}(R\bar{A}_2 + M_3 \bar{B}_2) - \frac{1}{4}\frac{x_c F}{z_0 z_3}(RA_2 + M_1 B_2),$$

where $M_1 = M - x_c$, $M_3 = 3M - x_c/3$. The values for x_0, x_2, and x_4 are now substituted in Eq. (9). However since $\bar{A}_2$, $\bar{B}_2$, $\bar{A}_4$, and $\bar{B}_4$ are negligible only the first harmonics of x_0, x_2, and x_4 are substituted in Eq. (9). Carrying out the integration for the steady-state solution and simplifying we obtain for the value of x_6 as far as the first harmonic

$$x_6 = V^7 A_6 \sin(\xi - \phi) + V^7 B_6 \cos(\xi - \phi),$$

where

$$A_6 = -\frac{\mathrm{I}}{8z_0}[RF'(6T^2 + 18S^2 + 18D_0 + 18Q) + x_c F(12ST + 6W) + (25RG'S + 5x_c GT)],$$

$$B_6 = +\frac{\mathrm{I}}{8z_0}[x_c F(6T^2 + 18S^2 + 18D_0 + 18Q) - RF'(12ST + 6W) + (25x_c GS - 5RGT)],$$

where

$$S = \frac{\mathrm{I}}{z_0}(RA_2 + M_1 B_2), \qquad T = \frac{\mathrm{I}}{z_0}(RB_2 - M_1 A_2),$$

$$Q = \frac{\mathrm{I}}{z_0}(RA_4 + M_1 B_4), \qquad W = \frac{\mathrm{I}}{z_0}(RB_4 - M_1 A_4).$$

The substitution of the values of x_0, x_2, x_4, and x_6 in Eq. (5) yields *a* steady-state solution of Eq. (4).

Natural Period of the Circuit. Let the maximum value of x in the operation of the circuit be denoted by c. It is supposed that the applied voltage maintains this maximum in the steady-state operation. We desire to obtain the natural period of the circuit for this mode of operation. That is, we desire a solution of Eq. (4) for $E = 0$. However, the natural period of Eq. (4) will first be obtained neglecting damping. The equation to be investigated is

$$M\frac{dx}{d\theta} + x_c(x + Fx^3 + Gx^5)d = 0,$$

or

$$M\frac{d^2x}{d\theta^2} + x_c(x - a_3x^3 + a_5x^5) = 0. \qquad (10)$$

Performing a first integration of Eq. (10) we have

$$\left(\frac{dx}{d\theta}\right)^2 = \frac{a_5 x_c}{3M}\left(\frac{3}{a_5}C_0^2 - \frac{3}{a_5}x^2 + \frac{3a_3}{2a_5}x^4 - x^6\right), \quad (11)$$

where

$$\left(\frac{dx}{d\theta}\right)^2 = \frac{x_c C_0^2}{M} \quad \text{for} \quad x = 0.$$

The period of the solution is dependent on the amplitude of the flux.

The right side of Eq. (10) vanishes for only one real value of x, i.e., at the maximum value of the flux. Accordingly, let (11) be written

$$\left(\frac{dx}{d\theta}\right)^2 = \frac{a_5 x_e}{3M}(c^2 - x^2)[x^4 + 2(b^2 - a^2)x^2 + (a^2 + b^2)^2] = 0, \quad (12)$$

where

$$\begin{gathered}(a^2 + b^2)^2 = \frac{3C_0^2}{a_5 c^2}, \qquad (a^2 + b^2)^2 - 2(b^2 - a^2)c^2 = \frac{3}{a_5}, \\ c^2 - 2(b^2 - a^2) = \frac{3a_3}{2a_5}, \qquad C_0^2 = c^2\left(1 - \frac{a_3}{2}c^2 + \frac{a_5}{3}c^4\right), \\ a^2 = \frac{1}{2}\left(c^4 - \frac{3}{2}\frac{a_3}{a_5}c^2 + \frac{3}{a_5}\right)^{1/2} - \frac{1}{4}\left(c^2 - \frac{3}{2}\frac{a_3}{a_5}\right), \\ b^2 = \frac{1}{2}\left(c^4 - \frac{3}{2}\frac{a_3}{a_5}c^2 + \frac{3}{a_5}\right)^{1/2} + \frac{1}{4}\left(c^2 - \frac{3}{2}\frac{a_3}{a_5}\right).\end{gathered} \quad (13)$$

Setting $x = cy$, $a/c = \beta$, and $b/c = \gamma$, Eq. (12) becomes

$$\frac{d^2y}{dt^2} = A_0^2 c^4(1 - y^2)[(\beta^2 + \gamma^2)^2 + 2(\gamma^2 - \beta^2)y^2 + y^4]. \quad (14)$$

The determination of the period from Eq. (14) now resolves itself into two cases.

Case (1). In this case $(\beta^2 + \gamma^2) > 1$. The period in this case, from Eq. (14), is

$$\begin{aligned}\bar{T}_e &= \frac{2}{c^2 \bar{A}_0(\beta^2 + \gamma^2)}\int_{-1}^{+1} \frac{dy}{\sqrt{(1 - y^2)(1 - \lambda_0^2 y^2 + \lambda^4 y^4)}}, \\ &= \frac{4}{c^2 \bar{A}_0(\beta^2 + \gamma^2)}\int_0^1 \frac{(1 - 2xh + h^2)^{-1/2}}{\sqrt{1 - y^2}}dy,\end{aligned}$$

where

$$h = \lambda^2 y^2, \qquad -1 \leq x = \frac{\lambda_0^2}{2\lambda^2} \leqq +1, \qquad \bar{A}_0^2 = \frac{a_5 x_c}{3M},$$

$$\lambda^4 = \frac{1}{(\beta^2+\gamma^2)^2}, \qquad \lambda_0^2 = \frac{-2(\gamma^2-\beta^2)}{(\gamma^2+\beta^2)^2}.$$

Since $0 < h < 1$, $-1 < x < +1$,

$$(1 - 2xh + h^2)^{-1/2} = 1 + hP_1(x) + h^2P_2(x) + \cdots,$$

where P_1, P_2, $\cdots$ are Legendre Polynomials. Substituting this value of the radical in the above integrand and carrying out the integration the value of T is

$$\bar{T}_c = \frac{2\pi}{c^2\bar{A}_0(\beta^2+\gamma^2)}\left(1 + \frac{\lambda_0^2}{4} + \frac{9\lambda_0^4}{64} - \frac{3\lambda^4}{16} + \cdots\right). \quad (15)$$

Integration of the Equation for Case 1; Damping Neglected. The differential equation is

$$\frac{d^2y}{d\theta^2} + \bar{A}^2[(1+\lambda_0^2)y - 2(\lambda_0^2+\lambda^4)y^3 + 3\lambda^4y^5] = 0, \quad (16)$$

where

$$x_c = \frac{\bar{x}_c(R_1+R_2)}{R_2}, \qquad \bar{A} = c^2(\beta^2+\gamma^2)\bar{A}_0.$$

Changing the independent variable from θ to t by the equation of transformation $\theta = \frac{\bar{Q}}{\bar{A}}t = \frac{(1+\delta)}{\bar{A}}t$ and replacing λ_0^2 by $\lambda_0^2\mu^2$ and λ^2 by $\lambda^2\mu^2$ we obtain

$$\frac{d^2y}{dt^2} + (1+\delta)^2[(1+\lambda_0^2\mu^2)y - 2(\lambda_0^2\mu^2+\lambda^4\mu^4)y^3 + 3\lambda^4\mu^4y^5] = 0. \quad (17)$$

The last equation is integrable as a power series in μ^2. Substituting

$$y(t) = y_0(t) + y_2(t)\mu^2 + y_4(t)\mu^4 + \cdots \quad (18)$$
$$\delta = \delta_2\mu^2 + \delta_4\mu^4 + \cdots$$

in Eq. (17) and equating like powers of μ^2 we have

$$y_0'' + y_0 = 0,$$
$$y_2'' + y_2 = -\lambda_0 y_0 + 2\lambda_0^2 y_0^3 - 2\delta_2 y_0, \quad (19)$$
$$\cdots\cdots\cdots\cdots$$

Before integrating sequentially Eqs. (19) it is necessary to determine the initial conditions in t. From Eq. (14), $\frac{dy}{d\theta} = \bar{A}$ for $y = 0$. By the change of variable

$$\theta = \frac{\bar{Q}}{\bar{A}}t, \qquad \frac{dy}{d\theta}\frac{\bar{A}}{\bar{Q}}\frac{dy}{dt} \quad \text{or} \quad \frac{dy}{dt} = \bar{Q} = \left(1 + \frac{\lambda_0^2\mu^2}{4} + \cdots\right).$$

The solution of the first equation of (19), subject to the initial conditions $y_0'(0) = 1$, and $y_0(0) = 0$, is $y_0 = \sin t$. The substitution of $y_0 = \sin t$ in the second of (19) yields

$$y_2'' + y_2 = \left(-2\delta_2 - \lambda_0^2 + \frac{3}{2}\lambda_0^2\right)\sin t - \frac{\lambda_0^2}{2}\sin 3t;$$

in order that this equation possess a periodic solution it is necessary that $\delta_2 = \frac{\lambda_0^2}{4}$. The solution, subject to the initial conditions $y_2'(0) = 0$, $y_2(0) = \frac{\lambda_0^2}{4}$, is

$$y = \frac{1}{16}\lambda_0^2(\sin t + \sin 3t).$$

This process can be carried as far as desired; the series solution

$$y = y_0(t) + y_2(t)\mu^2 + \cdots$$

converges for μ^2 sufficiently small or for $\mu^2 = 1$ and λ_0^2 and λ^2 sufficiently small. This solution is periodic of period 2π in t but of period $\bar{T}_c$ in θ.

Integration of the Equation for Case 1; Damping Included. If Eq. (4) is differentiated with respect to θ, E set equal to zero, use made of Eqs. (13), x set equal to cy, and the independent variable changed from θ to t by the relation $\theta = \frac{\bar{Q}t}{A}$, then the equation becomes

$$\frac{d^2y}{dt^2} + \bar{r}(1 + 3\bar{b}_3y^2 + 5\bar{b}_5y^4)\frac{dy}{dt} + \bar{Q}^2(\bar{b}_1'y + \bar{b}_3'y^3 + \bar{b}_5'y^5) = 0, \quad (20)$$

where

$$\bar{r} = \frac{R}{x_c}\bar{Q}\bar{A}b_1', \qquad b_1 = 1 + \lambda_0^2, \qquad b_3' = -2(\lambda_0^2 + \lambda^4),$$
$$b_5' = 3\lambda^4, \qquad b_3 = \frac{R_1 b_3'}{R b_1'}, \qquad b_5 = \frac{R_1 b_3'}{R b_1'}. \tag{21}$$

Equation (20) has been integrated in reference 1. The period of its solution is

$$T = \frac{2\pi}{\bar{A}}\bar{Q}\left(1 + \frac{r^2}{8}\delta_0\right), \tag{22}$$

where

$$\delta_0 = 1 + 2.24b_3e^2 + (2.08b_5 + 1.4b_3^2)e^4 + 0.36b_3b_5e^6 + 1.4b_2^2e^8, \tag{23}$$

where e^2 is in this case unity.

Integration of the Equation for Case 2; Damping Included. For the case $(\beta^2 + \gamma^2) \leqq 1$, Eq. (4) has been reduced in reference 2, by the steps indicated preceding Eq. (20), to the form

$$\frac{d^2y}{dt^2} + r(1 + 3b_3y^2 + 5b_5y^4)\frac{dy}{dt} + Q^2(b_1'y + b_3'y^3 + b_5'y^5) = 0, \tag{24}$$

where the values of the constants in case 2 are

$$r = \frac{R}{x_c}QAb_1, \qquad b_1' = k^4 - 2k_0^2, \qquad b_3' = 2(2k_0^2 - 1),$$
$$b_5' = 3, \quad b_3 = \frac{R_1}{R}\frac{b_3}{b_1}, \quad b_5 = \frac{R_1}{R}\frac{b_5}{b_1}, \quad x_c = \bar{x}_c\frac{(R_1 + R_2)}{R}, \tag{25}$$
$$k^2 = (\beta^2 + \gamma^2), \quad k_0^2 = (\gamma^2 - \beta^2), \quad R = \frac{\bar{x}_c B_0\alpha}{R_2} + R_1,$$

$A^2 = \frac{x_c a_5}{3M}$, and Q is given by the formulas found on page 566 of reference 2. The period T_{cr} in case 2 is

$$T_{cr} = \frac{2\pi}{A}Q\left(1 + \frac{r^2}{8}\delta_0\right), \tag{26}$$

where

$$\delta_0 = 1 + 2.24b_3e^2 + (2.08b_5 + 1.4b_3^2)e^4 + 0.36b_3b_5e^6 + 1.4b_5^2e^8, \quad (27)$$

and $e \doteq (\beta^2 + \gamma^2)$.

III. EQUIVALENT LINEAR CIRCUIT; DAMPING CRITERION.

Solution of the Complete Equation. If in Eq. (18), t is replaced by $t = \frac{A}{Q}\theta$ then

$$x = c[y_0(\theta) + y_2(\theta) + y_4(\theta) + \cdots] \quad (28)$$

is the solution of Eq. (4) for $E = 0$. Equation (5) is a steady-state solution of Eq. (4). Of course, the sum of Eqs. (5) and (28) is not the complete solution of Eq. (4) because (4) is non-linear. However, in all cases, when E is below (or even above) resonant voltage the flux is nearly sinusoidal. Thus the solution of the left side of (4) equated to zero is a sinusoid of period $\bar{T}_{cr}$ or T_{cr}. Accordingly, we shall replace the left member of Eq. (4) by the simple linear expression

$$\frac{d^2x}{d\theta^2} + h^2x, \quad (29)$$

where $\frac{1}{h} = \frac{2\pi}{T_{cr}}$ or $\frac{1}{h} = \frac{2\pi}{\bar{T}_c}$. A steady-state solution of (4) is (5) and thus the solution required is

$$x = A \sin h\theta + B \cos h\theta + x_0(\theta) + x_2(\theta) + \cdots. \quad (30)$$

We shall define the equivalent linear circuit to be the linear circuit given by equations (29) and (30).

Damping Criterion. From physical considerations it is evident that R_2 can be so small that the solution of Eq. (4) for $E = 0$ is not oscillatory. In fact the industrial importance of this paper is to determine the economical values of R_2 for which no beats exist, i.e., for which the solution is not given by Eq. (30). Equation (4), for $E = 0$, reduces to either Eq. (20) or (24). The solutions of Eq. (20) and (24) so far obtained assume that $\bar{r}$ and r are small. In Eqs. (20) and (24) denote the average values, taken with re-

spect to y from $y = 0$ to $y = 1$, of $(1 + 3\bar{b}_3y^2 + 5\bar{b}_5y^4)$ and $(1 + 3b_3y^2 + 5b_5y^4)$ by $\bar{N}$ and N respectively. Write Eqs. (20) and (24)

$$\frac{d^2y}{dt^2} + \bar{r}\bar{N}\frac{dy}{dt} + \bar{Q}^2(\bar{b}_1y + \bar{b}_3y^3 + \bar{b}_5y^5) = 0, \qquad (\beta^2 + \gamma^2) > 1 \quad (31)$$

$$\frac{d^2y}{dt^2} + rN\frac{dy}{dt} + Q^2(b_1y + b_3y^3 + b_5y^5) = 0, \qquad (\beta^2 + \gamma^2) \leqq 1. \quad (32)$$

Now the value for y is approximately sinusoidal. The criterion that the first term of the solution of (32) be not periodic is that

$$r^2N^2 - 4Q^2b_1 > 0. \qquad (33)$$

Equation (33) is the criterion desired. This criterion contains all the constants of the original circuit. If r, N, Q, and b_1 are expressed in terms of the constants of the circuit and the resulting equation solved for $\bar{x}_c$ as a function of R_2 a double valued function is obtained whose graph checks the results obtained on the differential analyzer by J. W. Butler and C. Concordia of the General Electric Company. (See Ref. 1.)

IV. DIFFERENTIAL ANALYZER CHECKS OF THE THEORY.

Value of c. The value of c cannot be obtained from Eq. (5) since this equation gives a steady-state solution under the assumption that there are no beats taking place. The value from Eq. (5) is much too small. However, the maximum value of x on the first half cycle is a good approximation to c in most cases. We shall indicate later when it is not a good approximation. To obtain the above value of c, consider the saturation curve (Fig. 3) to be made up of 2 or 3 straight lines. Let the common point of the first two straight lines l_1 and l_2 be at i_1. Solve the circuit concerned with the first straight line by Heaviside's Operational Calculus. Obtain the value of θ_1 corresponding to i_1 in this solution. Next compute the charge q_1 on the condenser up to the time θ_1. Using the initial conditions i_1, q_1, and θ_1 as initial conditions solve the next circuit in which the inductance is the slope of l_2.

By continuing this process (2 lines are usually sufficient) we obtain the maximum value of i for the first half cycle and consequently B and x.

There remains the unstable case. Employing the value of c obtained above compute the period by either equation (22) or (26). If this period is a near integral multiple of the applied voltage period and if in addition the condenser voltage is in unison with the applied voltage at the ends of the integral multiple period, the flux will build up with increasing time. But from the analysis the damping increases as the maximum value of the flux increases. This may set a limiting value. At any rate the current will at most be resistance limited. (See Ref. 2.)

Saturation Curve. The theory is now applied to representative transmission line circuits. If E is such that the maximum value of the total flux B is above the knee of the curve (Fig. 3) then Stirling's formula gives an exceedingly close approximation to the saturation curve. The difference table for $B = \psi + L_1$ and for maximum $B = 1.65$ (curve 1, Fig. 3(b)) is

B	i							
−1.65	−1.44							
		1.38						
−1.10	− .06		−1.3248					
		.0552		1.2744				
− .55	− .0048		− .0504		−1.224			
		.0048		*.0504*		*1.224*		
.00	0		0		0			
		.0048		*.0504*		*1.224*		
.55	.0048		.0504		−1.224			
		.0552		1.2744				
1.10	.0600		1.3248					
		1.38						
1.65	1.44							

Whence

$$i = .0048x + \frac{.0504}{6}(x^3 - x) + \frac{1.224}{120}x(x^2 - 1^2)(x^2 - 2^2),$$

or

$$27.8i = x - 1.14x^3 + 0.284x^5,$$

where $x = \dfrac{B}{B_0}$ and $B_0 = 0.55 = 1/3$ the maximum value of B.

If E is so small that the maximum value of B is near the knee of the curve then B_0 is the slope of the saturation curve through the origin. This slope must be read from Fig. 2 out to about $B = 0.6$. Two points on the curve then give a_3 and a_5.

Table. The following table displays the computations for the circuits of Figs. 4, 5, 6, 7, and 8. The entries fourteen to twenty-four, for cases 3, 4, 6, and 7, are values for symbols without the bar.

	R_1	R_2	$\bar{x}_c$	R	x_c	B_0
1. Fig. 4. Curve (a)	0.2	6.3	0.35	2.66	0.37	0.53
2. Fig. 4. Curve (b)	0.2	6.3	0.35	2.66	0.37	0.53
3. Fig. 5. Curve (a)	0.2	6.7	0.21	0.68	0.216	0.60
4. Fig. 5. Curve (b)	0.2	4.2	0.21	0.83	0.22	0.60
5. Fig. 6	0.049		0.21	0.049	0.21	76.00
6. Fig. 7	0.049		0.21	0.049	0.21	0.55
7. Fig. 8	0.049		0.21	0.049	0.21	0.55

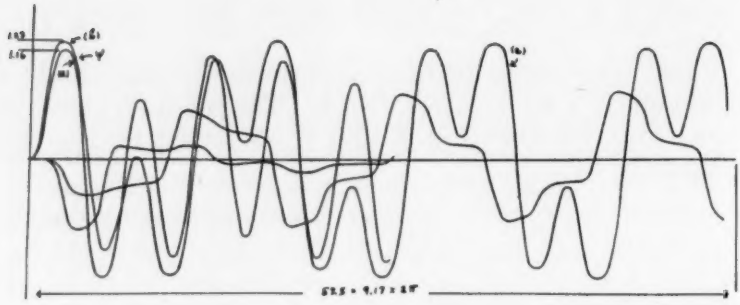
	L	α	a_3	a_5	c	β	γ
1.	0.4	83.5	− .025	.31	2.70	.55	.89
2.	0.4	83.5	− .025	.31	3.02	.52	.88
3.	0.4	7.85	.264	.053	3.00	.65	.714
4.	0.4	7.85	.264	.053	3.00	.65	.714
5.	0.2	1.0	-1.21×10^{4}	1.87×10^{8}	1.52×10^{-2}	.80	1.39
6.	0.2	27.8	1.41	.284	2.74	.47	.47
7.	0.2	27.8	1.41	.284	3.18	.49	.60

	$\bar{A}$	$\bar{T}_c$	$\bar{r}^2$	$1+(\bar{r}^2\delta_0/8)$	$\bar{T}_{cr}$	$\bar{T}_t$	$\bar{Q}$
1.	.23	$3.43\times2\pi$	$(.22)^2$	1.028	$3.52\times2\pi$	$3.58\times2\pi$	.79
2.	.29	$2.68\times2\pi$	$(.118)^2$	—	—	—	.77
3.	.257	$2.64\times2\pi$	$(.386)^2$	1.28	$3.38\times2\pi$	$3.05\times2\pi$	.678
4.	.258	$2.63\times2\pi$	$(.465)^2$	1.414	—	—	.678
5.	.0695	$13.5\times2\pi$	$(.0131)^2$	1.000	$13.5\times2\pi$	$13\times2\pi$	.94
6.	.267	$4.45\times2\pi$	$(.0151)^2$	1.0001	$4.45\times2\pi$	$5\times2\pi$	1.19
7.	.359	$2.22\times2\pi$	$(.0132)^2$	1.0024	$2.22\times2\pi$	$3\times2\pi$	.794

	$\bar{b}_1$	$\bar{N}$	$\bar{r}^2\bar{N}^2$	$4\bar{Q}\bar{b}_1$	Nature
1.	.17	1.18	.210	.42	Unstable
2.	.073	3.9	.213	.172	Stable
3.	.705	2.65	1.04	1.30	Unstable
4.	.705	2.65	1.52	1.30	Stable
5.	.86		0+	3.05	Very unstable
6.	.204	5.9	.008	1.16	Very unstable
7.	.20	12.6	.028	.504	Very unstable

In the above table T_t denotes the test period obtained from the curves of the differential analyzer. The quantities $\bar{N}$ and N are respectively $1 + \bar{b}_3 + \bar{b}_5$ and $1 + b_3 + b_5$.

FIG. 4.

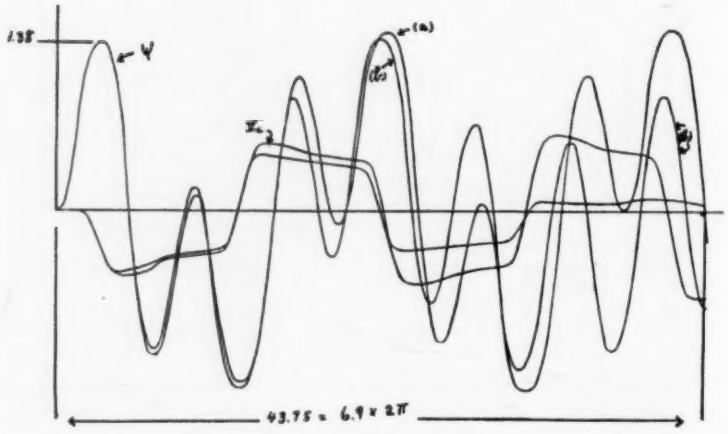


Saturation curve 2.

$R_2 = 6.3$
$R_1 = .197$
$L = .393$
$\bar{x}_0 = \frac{I}{C} = .35$

(a) $E = .77$
(b) $E = 1.00$
$e = E \sin (\theta - \theta_0)$
$\theta_0 = 0$

FIG. 5.



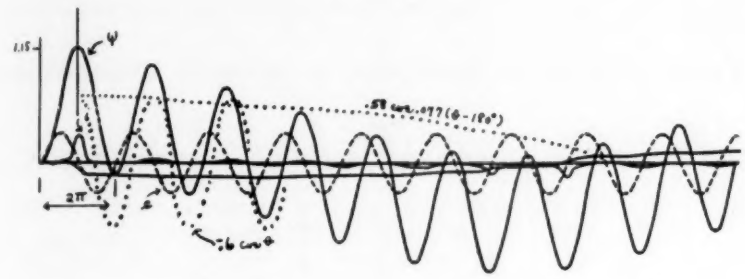
Saturation curve 1.

$L = .394$
$R = .197$
$\bar{x}_0 = \frac{I}{C} = .21$
$e = E \sin (\theta - \theta_0)$
$\theta_0 = 0$

(a) $R_2 = 6.7$
(b) $R_2 = 4.2$

Calculation of ψ Wave. Since the procedure is the same in every case we shall calculate the wave only for Fig. 6. A steady-state flux by Eq. (5) ($m^2 = 1$) is $0.6 \sin (\theta - 90°)$,

FIG. 6.

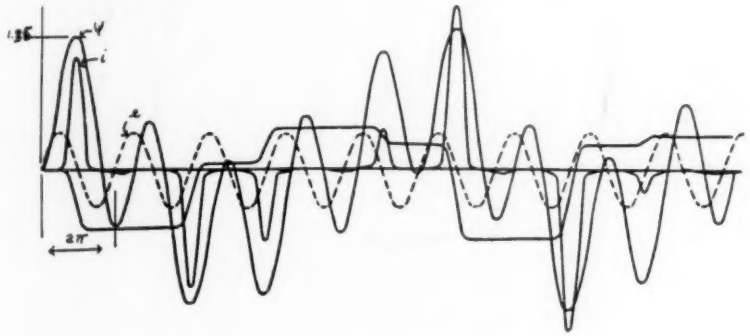


Saturation curve 1.

$E = .6$
$R = .05$
$L = .197$
$\bar{x}_c = \frac{1}{c} = .217$

$\theta_0 = 0$
$e = E \sin (\theta - \theta_0)$

FIG. 7.



Saturation curve 1.

$E = .75$
$R = .05$
$L = .197$
$\bar{x}_c = \frac{1}{c} = .214$

$e = E \sin (\theta - \theta_0)$
$\theta_0 = 0$

all higher harmonics being negligible. Expression (29) is $\frac{d^2B}{d\theta^2} + (.077)^2 B$. Equation (30) is

$$B = C_1 \sin (.077\theta) + C_2 \cos (.077\theta) - 0.6 \cos \theta.$$

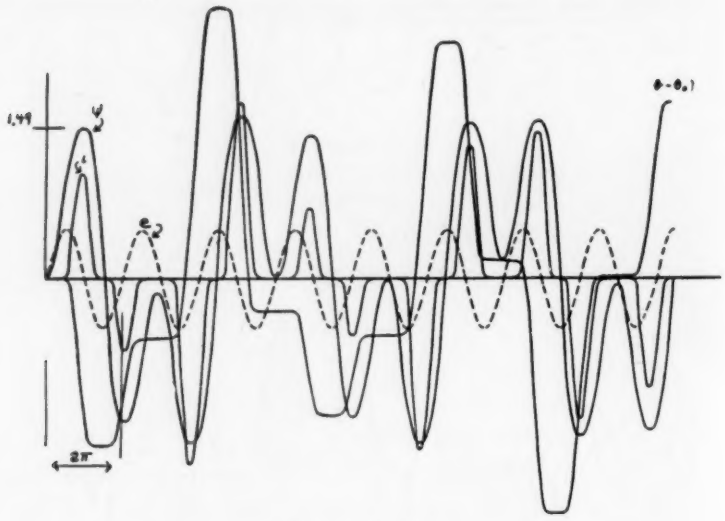
The value of c as obtained from the first paragraph of Part IV is 1.2 and the displacement angle is π. We thus have

$$B = 1.2 = C_1 \sin .077\pi + C_2 \cos .077\pi - 0.6 \cos \pi,$$

$$\frac{dB}{d\theta} = 0 = .077C_1 \cos .077\pi - .077C_2 \sin .077\pi + 0.6 \sin \pi.$$

Whence $B = .58 \cos .077(\theta - \pi) - 0.6 \cos \theta$. ψ is obtained

FIG. 8.



Saturation curve 1.

$E = 1.00$
$R = .05$
$L = .197$
$\bar{x}_c = .214$

$e = E \sin (\theta - \theta_0)$
$\theta_0 = 0$

from B by inspection of the graphs of Fig. 3. In this example $\psi \doteq B$. This equation holds for θ greater than π. The two curves $.58 \cos .077(\theta - \pi)$ and $- 0.6 \cos \theta$ are shown in Fig. 6. By inspection, their sum gives very accurately the curve of the differential analyzer.

Conclusion. While the curves of Figs. (4, 5, 6, 7, and 8) are representative of industrial applications, the first four curves are critical test cases to test the accuracy of the theory. The criterion is valid on even these critical cases.

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Nylon.—Four patents for hosiery and other knitted fabrics made of nylon, the new organic textile fiber derived basically from coal, water and air, were issued to E. I. Du Pont de Nemours & Company by the U. S. Patent Office recently. With the issuance of the patents, officials of the Company asserted the inventions represented "a phenomenal advance in the textile arts," assuring "a truly successful stocking from a synthetic yarn." The patents set forth that nylon has great elasticity, strength and wearing qualities; that it presents an excellent appearance; that it is virtually wrinkle free; that it dries extremely rapidly after laundering; and that stockings made of it may be washed repeatedly without material change in their original shape and smooth characteristics. The patents cover the use of polyamide yarn for stockings, pre-boarding of stockings, setting yarns and fabrics, and the use of sodium sulfite for setting. "Nylon is little sensitive to water spotting," says the Company announcement. "Stockings made of it are essentially crease-proof under ordinary conditions." The Company's chemists say nylon is the first man-made organic textile fiber prepared wholly from raw materials from the mineral kingdom. Other textile fibers in general use, including cotton, silk, wool, rayon, and linen, are derived from animal or vegetable materials. Nylon differs from rayon, another man-made fiber, in that it contains no cellulose. The word "nylon" was coined by the du Pont Company as the generic name for all materials defined scientifically as "synthetic fiber-forming polymeric amides having a protein chemical structure; derivable from coal, air, and water, or other substances, and characterized by extreme toughness and strength and the peculiar property to be formed into fibers and into various shapes, such as bristles and sheets." It is a most important discovery coming from chemical research and is so versatile that the extent of its varied commercial uses can only be guessed at today.

R. H. O.

GRAPHICAL ANALYSIS OF TRANSIENT PHENOMENA IN ELECTRIC CIRCUITS.*

BY

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Transient changes of current and voltage exert an important influence on the performance and safety of electric circuits and therefore have to be considered in their design and operation. For this reason this group of phenomena have been subjected to intensive study, both algebraically and experimentally. The algebraic method has been developed at the hands of Steinmetz (method of complex variable) and Heaviside (operational calculus) to an effective aid in predicting the behavior of a certain electrical circuit under certain conditions, but a considerable study is required to master the procedure, and they are not apt to convey a clear visualization of the events which take place. The experimental method, by means of oscillographic measurement, is applicable only to circuits actually in existence; it is useful for checking the validity of the conclusions of theory, but it does not allow the prediction of the behavior of a certain system in the design stage.

The present paper deals with a graphical method which has the advantage of easy applicability and clarity, enabling one to visualize the interplay of currents and voltages, and to extend the conclusions to a whole group of systems having a variety of component parts.

From the point of view of the graphical analysis the electric transients are a species of the broader family of transient phenomena in elastic columns, such as hydraulic conduits and mechanical springs. These formed the subject of previous treatises by the author. As it will be shown the elements of electric circuits bear an analogy to elements of hydraulic and mechanical systems, and their analysis is

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basically similar. An endeavor is made to make this paper self-contained and understandable without recourse to other literature; nevertheless for those desirous to acquaint themselves with the broader group of phenomena the study of the references given will be of interest.

REPRESENTATION OF ELECTRICAL PHENOMENA.

The electrical power p passing through a certain cross-section x of a circuit (x being the distance as measured from a point of reference) and at a certain instant t of time (as measured from the beginning of time-reckoning) can be expressed as the product of two components: the voltage e (electrostatic component) and the current i (electromagnetic component), that is:

$$p = ei.$$

Thus any "state of power" can be represented in the $i - e$ coördinata system by a point.

Considering first d.c. currents, the value of both the current and voltage may change from instant to instant and from point to point. Thus for the complete description of a phenomenon the knowledge of a pair of functions,

$$\begin{cases} i = f_1(t, x), \\ e = f_2(t, x), \end{cases}$$

is necessary. In graphical terms this pair of functions can be represented by a pair of stereograms with the i and e values, as vertical ordinates erected over the horizontal $t - x$ plane as base.

To every condition of circuit (value of impressed voltage and absorbed current) corresponds a "steady" state in which the current and voltage are either constant or are functions of the distance only:

$$\begin{cases} i = f_1(x), \\ e = f_2(x). \end{cases}$$

If, now, the original condition of the circuit is altered the power will assume a new state. In general, however, the change of state will not be instantaneous but a certain time will elapse before the new steady state will be attained, even if the circuit were altered instantaneously. The changes of

current and voltage during this interval of time, between the two steady states, are called "transient." In general the transient states will be functions of both time and distance,

$$i = f_1(t, x) \qquad \text{and} \qquad e = f_2(t, x),$$

and for their representation the above described pair of stereograms is necessary. In particular cases, however, the distance-variable can be neglected and then the transient is a function of the time only,

$$i = f_1(t) \qquad \text{and} \qquad e = f_2(t),$$

and can be represented by a pair of plane curves.

During the transient phenomenon the current and voltage assume a number of successive values each of which can be represented by points in the $i - e$ diagram. The succession of these points, which may be disconnected points, or contiguous points forming lines or areas, will be referred to as "surge-diagram." This serves as a connecting link between the i and e stereograms, and forms the main basis for the graphical method of analysis, as it will be shown in the later examples.

For many purposes the determination of the complete stereogram is not necessary and some significant sections of it are sufficient. Thus we will refer to the plan view of the stereograms, i.e.

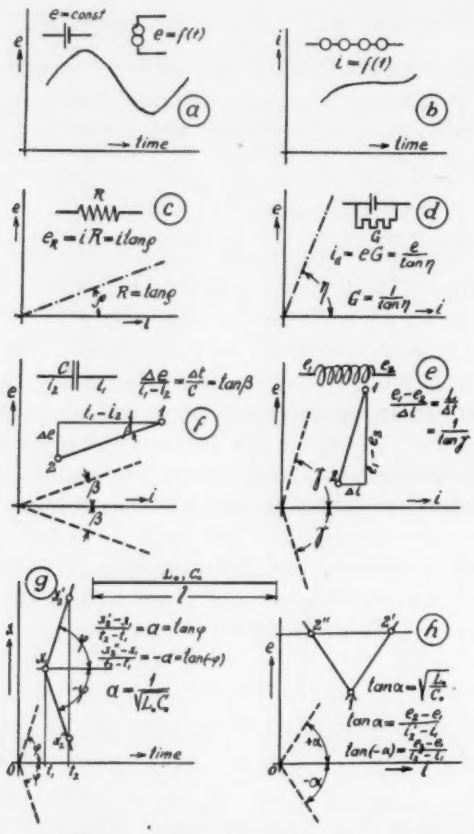
$$x = f(t),$$

as the "disturbance diagram" because it shows the progress of the disturbance-front as a function of time. The vertical sections taken parallel with the t-axis will be called the "history-diagram" of current and voltage, $i = f_1(t)_{x=x_1}$, $e = f_2(t)_{x=x_1}$. The vertical sections taken parallel with the x-axis will be called "gradient diagram" of current and voltage, $i = f_1(x)_{t=t_1}$, $e = f_2(x)_{t=t_1}$.

Similar considerations apply to a.c. or fluctuating currents. The state of such currents is called "steady" if their value of current and voltage varies periodically between the same limits according to the same law. Also in this case transient phenomena occur if the original condition of the circuit is changed. The graphical representation is analogous to that explained above for d.c. currents.

Elements of Electric Circuits and their Graphical Representation. (Fig. 1.)

FIG. 1.



Elements of Electrical Circuit. (*a*) Generator (d.c. or a.c.); (*b*) Consumer; (*c*) Resistance; (*d*) Leakance; (*e*) Inductance; (*f*) Capacitance; (*g*) Velocity of Propagation of a Disturbance in a Conduit; (*h*) Proportionality of Current-Change to Voltage-Change in a Conduit.

Every electric circuit can be considered as a combination of

(*A*) Power-generating elements: Battery, D.C. and A.C. Generator;

(*B*) Power-absorbing elements: Resistance, Leakance, Electric Motor;

(*C*) Energy-storing elements: Inductance, Capacitance;

(*D*) Power-transmitting element: Conduit.

The properties and characteristics of these, and their graphical representation will be treated in the following.

(1) The Battery is a source of a constant e.m.f.; it is represented in either the $i - e$, or the $t - e$, or the $x - e$ coördinate system by an $e = \text{const.}$ line. In a hydraulic system the analogous element is a storer of infinite capacity. The d.c. generator is a source of constant but alterable (e.g. by altering the speed) e.m.f.; it is represented by an $e = f(t)$ curve. The a.c. generator is a source of periodically variable e.m.f.; it also is represented by an $e = f(t)$ curve.

(2) The power-absorbing element can be represented in general by the $i = f(t)$ curve.

(3) The Resistance is characterized by a constant ratio of voltage to current,

$$R = \frac{e_R}{i} = \tan \rho; \qquad e_R = iR = i \tan \rho,$$

and it is represented in the $i - e$ diagram by a straight line forming an angle ρ with the i-axis. In a hydraulic system the analogous element is the orifice or nozzle.

(4) The Leakance is characterized by a constant ratio of current to voltage:

$$G = \frac{i_G}{e} = \frac{1}{\tan \eta}; \qquad i_G = eG = \frac{e}{\tan \eta},$$

and it is represented in the $i - e$ diagram by a straight line forming an angle η with the i-axis. In a hydraulic system the analogous element is a leakage area.

(5) The Electric Motor is characterized by the interrelationship between its terminal voltage and current and can be represented in the $i - e$ diagram by a corresponding curve.

(6) The Inductance is a storage for electromagnetic energy and it is characterized by the ratio of terminal-voltage to the rate of current-change, according to the equation

$$e_L = L\frac{di}{dt}; \qquad \frac{\Delta t}{L} = \frac{\Delta i}{e_L} = \frac{1}{\tan \gamma},$$

where L is the coefficient of inductivity, or of energy-storage

by current. The inductance can be represented in the $i - e$ diagram by sloping straight lines forming an angle γ with the i-axis. The interpretation of this angle γ is the following: If a potential difference e_L is acting on the terminals of the inductance, and the initial state of power is 1 $(= i_1, e_1)$ then, after the elapse of Δt time the current will change to $i_1 + \Delta i = i_2$ corresponding to point 2 situated on a tan γ directrix drawn from the 1 point. The analogous element in a hydraulic system is the "swinger"; in a mechanical spring system a concentrated mass.

(7) The Capacitance is a storage for electrostatic energy and it is characterized by the ratio of current to the rate of voltage-change, according to the equation

$$i_C = C\frac{de}{dt}; \qquad \frac{\Delta t}{C} = \frac{\Delta e}{i_C} = \tan \beta,$$

where C is the coefficient of capacity or energy-storage by voltage. The capacitance is represented in the $i - e$ diagram by sloping straight lines forming an angle β with the i-axis. The interpretation of this angle β is the following: If a current flows into (or out of) a capacitance and the initial state of power is 1 $(= i_1, e_1)$ then, after the elapse of Δt time the terminal voltage of the capacitance will change to $e_1 + \Delta e = e_2$ corresponding to point 2 situated on a tan β, directrix drawn from the 1 point. The analogous element in a hydraulic system is a storer; in a mechanical spring system a massless spring.

(8) The Conduit is an element in which inductivity, capacity, resistivity and leakage are present in distributed form. For the purposes of graphical analysis, however, we shall divest the conduit of its resistance and leakage and add these to the circuit in concentrated form. In this interpretation the conduit is endowed only with inductivity and capacity and thus capable of storing energy in both electromagnetic and electrostatic forms. It can be written

$$\left.\begin{aligned} \frac{\partial e}{\partial x} &= L_0\frac{\partial i}{\partial t}, \\ \frac{\partial i}{\partial x} &= C_0\frac{\partial e}{\partial t}, \end{aligned}\right\} \qquad \text{whence:} \quad \begin{aligned} \frac{\partial e}{\partial i} &= \sqrt{\frac{L_0}{C_0}}, \\ \frac{\partial x}{\partial t} &= \sqrt{\frac{1}{L_0 C_0}}, \end{aligned}$$

or, for finite differences:

$$\frac{\Delta e}{\Delta i} = \sqrt{\frac{L_0}{C_0}} = \tan(\pm \alpha) = \text{const.},$$

$$\frac{\Delta x}{\Delta t} = \sqrt{\frac{1}{L_0 C_0}} = a = \tan(\pm \varphi) = \text{const.}$$

The graphical interpretation of these equations is:

(*a*) In the $e - i$ chart the points 1 and 2, defining the state of power in the conduit before the disturbance (i_1, e_1) and after the disturbance (i_2, e_2) lie on a straight line forming a slope tan ($\pm \alpha$) with the i-axis. The slope is positive if both i and e either increase or decrease, i.e., by the disturbance power is supplied to, or absorbed from the conduit; the slope is negative if i increases while e decreases, or vice-versa, which is the case if by the disturbance no energy is supplied to the conduit. The expression $\sqrt{\frac{L_0}{C_0}}$ is called the "natural impedance" of the conduit.

(*b*) In the $t - x$ chart the progress of disturbance is defined by a straight line forming a slope tan ($\pm \varphi$) with the t-axis. The slope is positive if the disturbance is traveling away from the point of origin; it is negative if the disturbance is traveling towards the point of origin. The value a is the speed of propagation of the disturbance. The duration of time T which it takes the disturbance to travel from one end of the conduit to the other is called the "interval of single traverse" or briefly the "interval": $T = l/a = l\sqrt{L_0 C_0}$.

The analogous element in a hydraulic system is the pipe filled with liquid, i.e. the liquid column; in a mechanical system a spring or elastic rod having distributed mass.

In the foregoing characterization and representation of circuit-elements tacit approximations have been made. Only the conduit is assumed to have length, the other elements are assumed to be concentrated in one point in the circuit. Actually all physically realizable elements have a length dimension, and are adulterated by the admixture of other elements. Thus a resistance coil has some inductivity and capacity; the inductance has also resistivity and capacity; the

capacitance has leakage inseparably associated with it; a conduit has also resistivity and leakage besides its inductivity and capacity. In order to make the graphical analysis practically possible (in fact these approximations apply also to the algebraic analysis) the dominant features are emphasized, the secondary features are neglected, or assumed to exist in the circuit separated from the element instead of being inherently associated with it.

A further basic approximation inherent in the graphical analysis is the use of finite time-elements during which a certain state of power is assumed to be valid. Actually the state of power may, and in most cases does, change in a continuous instead of abrupt fashion; therefore the time-element of validity is infinitely small.

CLASSIFICATION OF ELECTRICAL TRANSIENTS.

In the absence of energy-storing elements any change in the condition of the circuit instantaneously affects the state of power passing through the circuit. In this case, therefore the transient phenomenon has zero duration.

Only in case there is an energy-storing element in the circuit can the transient phenomenon have a finite, or infinitely long, duration. According to the kinds of energy which can be stored in the circuit one-energy and two-energy transients can be distinguished. If in addition to the inductivity and capacity there is also a storage for kinetic energy (e.g. the rotor of the generator or motor) then it is capable of producing a three-energy transient.

According to whether the distance-variable is negligible or not one can distinguish transients in time only, and transients in time and distance.

According to the number of elements forming the circuit one-, two-, three- (and so on) element transients can be distinguished. In the following examples this last basis of classification will be freely followed, proceeding from simpler cases to more complicated ones.

EXAMPLES OF TRANSIENTS.

With the knowledge of the graphical representation, in the various combination of coordinates, of the components

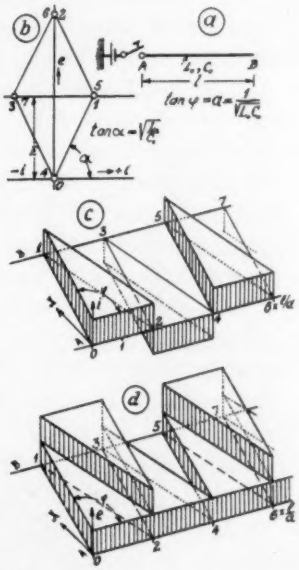
forming the electric circuit we are able to proceed to the solution of definite problems. From among the infinite number of possible transient phenomena we shall select only some simple combinations. In each case there shall be assumed as known: (1) the composition of the circuit, (2) the initial steady state and (3) the nature of disturbance. We shall seek to determine: (1) the stereograms of current and voltage, or (2) significant sections of these stereograms, and (3) the final steady state.

As to the nature of disturbance we shall deal only with the simplest kind, consisting of closing and opening of a switch.

1. Battery—Switch—Conduit. (Fig. 2.)

A free conduit is assumed, having an inductivity L_0 and capacity C_0 and zero resistivity, initially at zero potential.

FIG. 2.



Battery—Switch—Conduit. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) Stereogram of Current; (*d*) Stereogram of Voltage.

This initial state is represented by point o in the surge-diagram. At the beginning of our time-reckoning to one end of this conduit a constant voltage E is applied by closing the

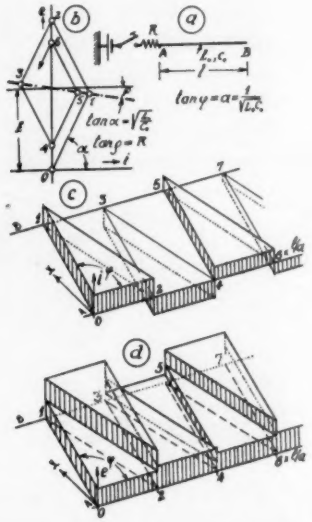
switch. It is assumed that the applied voltage remains constant and uninfluenced by the current drawn from it. The change of potential from zero to E is accompanied by a change of current the new state being represented by point 1 situated on the directrix tan α drawn from the 0 point. When this wave reaches the free end of the conduit the current is reduced to zero and the voltage is increased along the directrix 1–2 the point 2 being situated at the $i = 0$ line. The electromagnetic energy is changed into electrostatic energy. The new wave 2 returns towards the battery; on reaching it its voltage is reduced to E and the current reversed the new state being represented by the point 3 situated at the intersection of the 2–3 directrix with the E = const. line. The wave 3 on reaching the free end has its current reduced to zero and the new state will be represented by the point 4 being at the intersection of the 3–4 directrix with the $i = 0$ line. Then the play is repeated, and in the absence of energy-dissipating elements it would continue indefinitely. Erecting the i and e coördinates of the 1, 2, 3, 4, $\cdots$ and so on points on the $t - x$ chart as a base the current-stereogram Fig. 1*c* and the voltage-stereogram Fig. 1*d* are constructed which represent the current and voltage values for all points of the conduit and for all instants of time. From these it is easy to visualize the time-history of current and voltage at a given point x in the conduit by forming the intersection of these stereograms with the x = const. plane. Similarly, the intersections with a t = const. plane will give the values of current and voltage along the whole circuit at a certain instant. These stereograms clearly show that at the battery-end of the conduit the voltage is constant ($= E$) and the current alternates between $+ i$ and $- i$ values the period of the alternating cycles being $4l/a$; at the free end the current is zero at all times, but the voltage fluctuates between 0 and $2E$ values. At the intermediate points of the circuit the current fluctuates between $+ i$, 0 and $- i$, and the voltage between 0, E and $2E$ values.

Battery—Switch—Resistance—Conduit. (Fig. 3.)

The resistance is represented by the straight line having the slope tan ρ drawn from the $i = 0$, $e = E$ point. The

vertical distance between the R-line and the E = const. line corresponds to the voltage-drop e_R due to the resistance, $e_R = iR = i \tan \rho$. The initial state of the conduit is represented by the point 0 ($i = 0$, $e = 0$). On closing the switch the first disturbance wave starts at the A point of the conduit the new state being defined by the point 1 situated at the intersection of the directrix 0–1 drawn from the 0 point and the R-line. On reaching the free end of the conduit the current

FIG. 3.



Battery—Switch—Resistance—Conduit. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) Stereogram of Current; (*d*) Stereogram of Voltage.

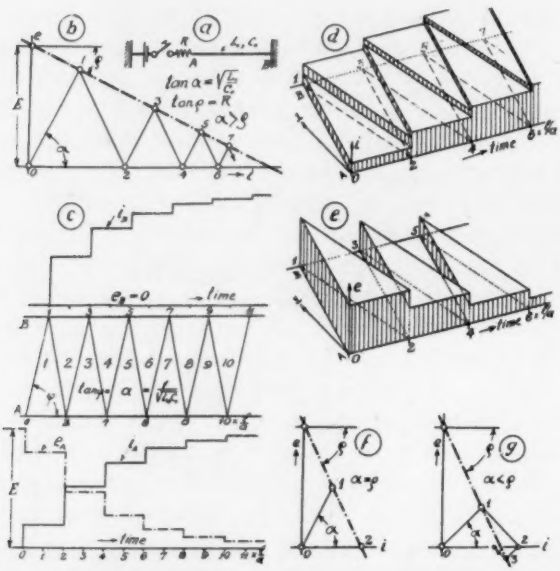
is reduced to zero and the voltage increases, the new state being defined by point 2 in the surge-diagram. This new wave returns towards the battery where the voltage is reduced to E plus the excess amount necessary to overcome the resistance, and the current is reversed, point 3, and on reaching the free end of the conduit the current is reduced again to zero, point 4. This process is repeated again, with successive waves having alternate current values at the battery-end A, and fluctuating voltage-values at the free end B. Due to the energy dissipation of the resistance these fluctuations lose amplitude at each successive wave in a

geometric progression, until after the elapse of an infinite number of waves the final, steady state with $e = E$ and $i = 0$ is attained. The stereograms of current and voltage are shown in the Fig. 3*c* and *d* respectively.

Battery—Switch—Resistance—Conduit—Ground. (Fig. 4.)

The surge diagram, Fig. 4*b* shows the representation of the conduit, the directrix tan α and of the resistance, $R = \tan \rho$.

FIG. 4.



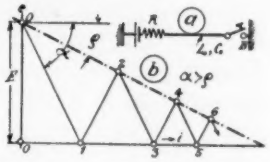
Battery—Switch—Resistance—Conduit—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at the Two Ends of Conduit; (*d*) Stereogram of Current; (*e*) Stereogram of Voltage; (*f*) Critical Resistance; (*g*) Greater Than Critical Resistance.

The state of power at the resistance-end of the conduit is characterized by a point situated at the resistance line, tan ρ, and the state of electric power at the ground-end of the conduit is characterized by a point situated at the abscissa-axis. The points defining the current and voltage of the successive waves are found by intersecting these two lines with the tan $(\pm \alpha)$ directrices, in the manner shown on Fig. 4*b*. The states i and e at the two ends of the conduit are of special interest. These are shown on Fig. 4*c* the middle portion of

which shows the location of the disturbance front as a function of time, traveling back and forth between the two ends of the conduit. For the triangular "zones" defined by these lines the same state of power, i and e are valid, as it is more clearly shown in the complete stereograms Fig. 4*d* and *e*. Depending on the value of resistance, tan ρ in relationship to the value of conduit characteristics tan α the transient phenomenon can be of various types. Fig. 4*f* shows the surge-diagram for the case of $\alpha = \rho$ in which the steady state, $i = E/\tan \rho$ and $e = 0$ is attained by the second wave, i.e. in a finite number of steps. Fig. 4*g* shows the surge-diagram for the case of $\alpha < \rho$, in which case at every fourth wave the voltage has a reversed sign, and again, the steady state is reached only in an infinite number of steps.

The position of the switch is not irrelevant for the progress of the transient. Fig. 5 shows the surge-diagram for the case

FIG. 5.



Battery—Resistance—Conduit—Switch—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram.

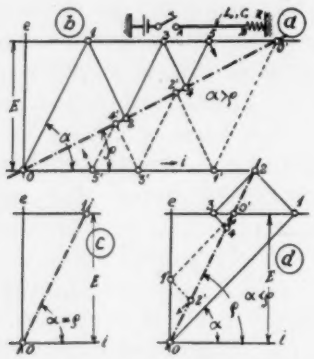
in which the switch is adjacent to the ground. As it is seen in this case the first wave is a return wave of $e = 0$ traveling toward the battery.

The location of the resistance has also an influence on the transient phenomenon. Fig. 6 is the surge-diagram for the case in which the switch is located at the battery-end of the conduit, and the resistance is located at the ground-end of the conduit. In this case the final, steady state of power in the conduit will be defined by $i = E/\tan \rho$ and $e = E$. The dash lines show the transient produced by the sudden opening of the switch, i.e. breaking of the circuit, resulting in the final, steady state $i = 0$, $e = 0$.

Figure 6*c* is the surge-diagram for the particular case $\rho = \alpha$ in which the new steady state is reached right with the first

disturbance wave, on breaking the circuit the original state $i = 0$, $e = 0$ is instantaneously attained.

FIG. 6.

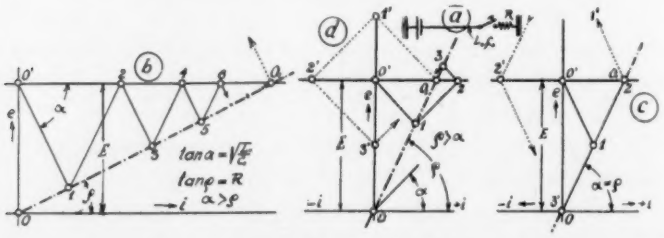


Battery—Switch—Conduit—Resistance—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram for Make and Break; (*c*) Critical Resistance; (*d*) Greater Than Critical Resistance.

Figure 6*d* is the surge-diagram for the case in which $\rho > \alpha$ for both the closure and the opening of the switch.

Figure 7 illustrates the case in which both the switch and the resistance are adjacent to the ground-end of the conduit.

FIG. 7.



Battery—Conduit—Switch—Resistance—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram for Small Resistance; (*c*) Surge-Diagram for Critical Resistance, Make and Break; (*d*) Surge-Diagram for Greater Than Critical Resistance, Make and Break.

Figs. 7*b*, *c* and *d* are the surge-diagrams for the case when $\alpha > \rho$, $\alpha = \rho$, and $\alpha < \rho$ respectively, which are self-explanatory. The transients on breaking the circuit are illustrated in dotted lines. As in this case the broken circuit does not contain the power-dissipating resistance therefore the current

and voltage oscillations persist in undiminished magnitude, for an indefinite time, after the breaking of the circuit.

Battery—Switch—Conduit—Inductance—Ground. (Fig. 8.)

As explained in the previous paragraphs the inductance is represented in the surge-diagram by straight lines with the slope tan γ for which

$$\tan \gamma = \frac{L}{\Delta t} = \frac{e}{\Delta i}.$$

The time-interval Δt is arbitrary; the smaller it is chosen the more exact the graphical construction will be, and the more laborious the procedure. The time-interval necessary for the traversal of the conduit from one end to the other is:

$$T = l/a = l\sqrt{L_0 C_0},$$

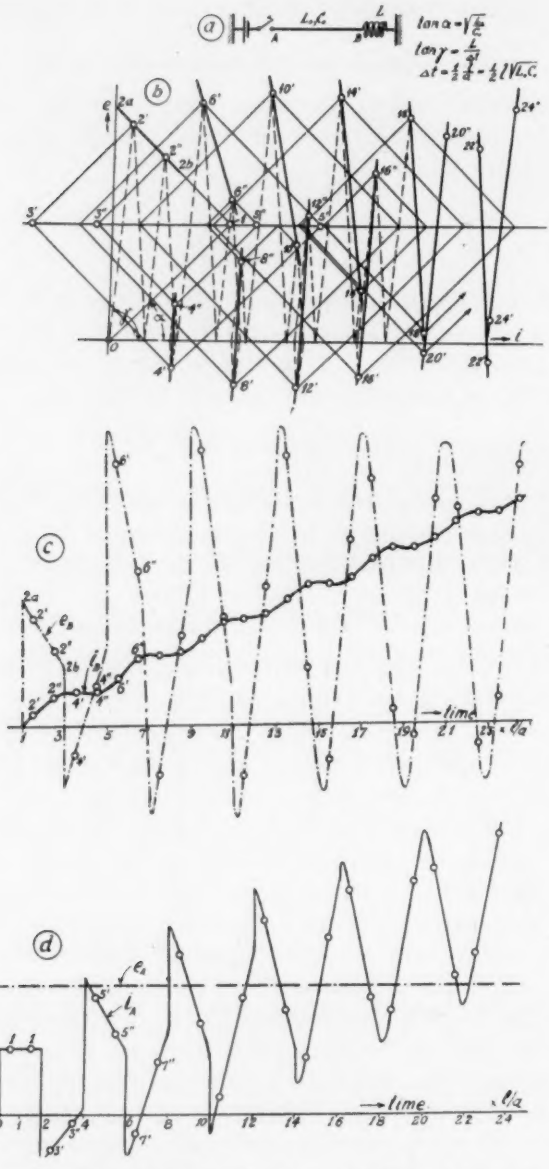
where L_0 and C_0 are the inductance and capacitance of the conduit. It is expedient to choose Δt equal to T or a sub-multiple of it. In this example we shall choose

$$\Delta t = T/2 = l/2a,$$

from which value the tan γ directrix can be determined and entered into the surge-diagram.

When the switch is closed at the beginning of time-reckoning at the switch-end of the conduit there will start a disturbance-wave the coördinates of which are defined by point 1. On reaching the inductance at the other end of the conduit the current is momentarily reduced to zero and the voltage doubled, according to point $2a$. After the elapse of Δt time the current will attain a finite value and the voltage will be reduced, according to point $2'$ located at the intersection of the tan γ directrix drawn from the 0 point (representing the increase of current as a function of potential difference) and the tan α directrix drawn from the 1 point (representing the change of current and voltage in the conduit). The increase of current and decrease of voltage continues till after the elapse of $3\Delta t$ time the state of power $2''$ and after the elapse of $4\Delta t$ time the state of power $2b$ is attained. Although the current and voltage are continually

FIG. 8.



Battery—Switch—Conduit—Inductance—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at the Battery-End of Conduit and (*d*) at the Inductance-End of Conduit.

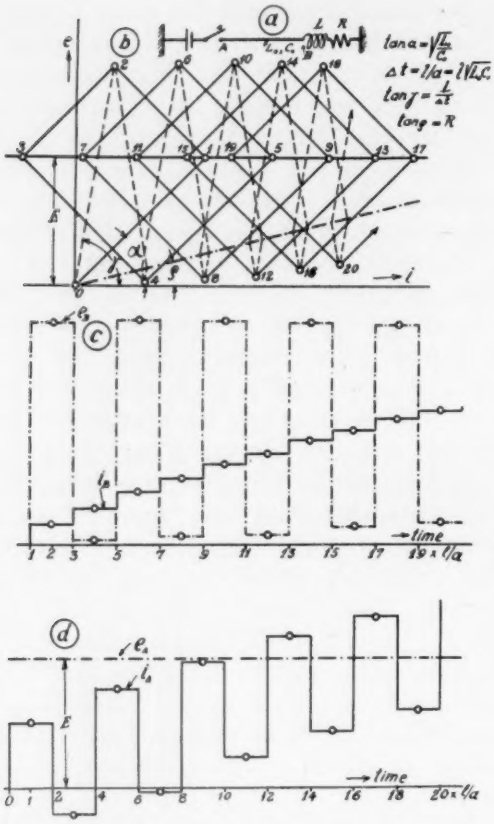
changing, for the sake of graphical construction we assume that during the first $2\Delta t$ interval the state $2'$ and during the second $2\Delta t$ interval the state $2''$ is valid and these waves are propagated towards the battery along the conduit. On reaching the battery the voltage E is re-established with a corresponding decrease, or even reversal, of the current, as defined by points $3'$ and $3''$ each of which is assumed to be valid for $2\Delta t$ intervals. These latter waves travel towards the inductance-end of the conduit, where the states $4'$ and $4''$ are found at the intersection of the tan α and tan γ directrices in the manner shown. The procedure is then continued as it is clearly shown on Fig. 8*b*. In this manner for each interval l/a two points are obtained defining the mean current and mean voltage in the first and second half of the interval. From these the stereograms can be constructed but they are omitted here and only the history of current and voltage for the switch-end (Fig. 8*c*) and for the inductance-end (Fig. 8*d*) are shown. It is emphasized that these values are approximative, because for full exactness the Δt should be chosen o while for the graphical analysis it must be chosen a finite quantity. If the fluctuations within an l/a period are of interest then the Δt interval should be chosen 1/3, 1/4 or a lesser submultiple of l/a. If, however, the fluctuations within each l/a period are of no particular interest, and only the trend of mean values is sought, then it is practicable to choose $t = l/a$ thereby saving in labor. This simplified procedure is shown for the circuit:

Battery—Switch—Conduit—Inductance—Resistance—Ground. (Fig. 9.)

The construction of the surge-diagram is clearly shown and does not need further explanation. At the battery-end of the conduit the voltage is constant, the current is fluctuating; at the inductance-end of the conduit the voltage is fluctuating, the growth of the current is gradual. The fluctuations have a diminishing trend and after the elapse of an infinite number of intervals the steady condition, corresponding to $i = E/\tan \rho$ and $e = E$ will be attained throughout the whole length of the conduit.

The sequence in which the various elements of the conduit are connected has an important influence on the character-

FIG. 9.



Battery—Switch—Conduit—Inductance—Resistance—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at Battery-End of Conduit and (*d*) at the Inductance-End of Conduit.

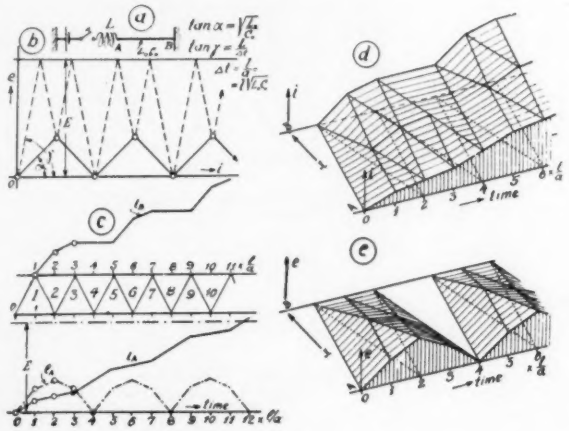
istics of the resulting transient, as it will be illustrated by the case:

Battery—Switch—Inductance—Conduit—Ground. (Fig. 10.)

The procedure of constructing the surge-diagram is shown clearly in Fig. 10*b*. The history of current and voltage at the inductance-end and at the ground-end of the conduit are shown in Fig. 10*c*; the middle of this shows the triangular $t - x$ zones in which the states defined by the points in the surge-diagram have validity. The complete stereograms for

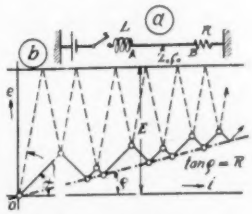
the conduit are given in Fig. 10*d* and *e* showing the change of current and voltage from point to point along the conduit and from instant to instant of time.

FIG. 10.



Battery—Switch—Inductance—Conduit—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at the Two Ends of the Conduit; (*d*) Stereogram of Current; (*e*) Stereogram of Voltage.

FIG. 11.



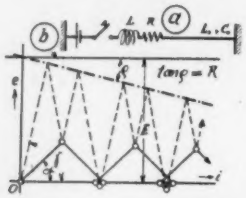
Battery—Switch—Inductance—Conduit—Resistance—Ground.
(*a*) Schematic Arrangement; (*b*) Surge-Diagram.

Figs. 11, 12 and 13, which are self-explanatory, show the surge-diagrams for various sequences of the circuit elements having also resistance.

The comparison of Fig. 4 and 5 with 12 and 13, and Figs. 6 and 7 with 9 and 11 shows that the final state of the conduit is the same in both sets of cases, but the presence of inductance retards the process, i.e. increases the number of intervals

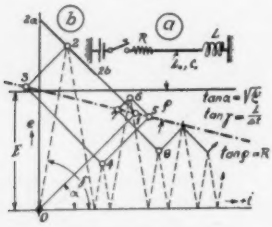
necessary for reaching a certain value of current. This fact is, of course, in full agreement with previous theoretical and experimental findings.

FIG. 12.



Battery—Switch—Resistance—Conduit—Inductance—Ground.
(*a*) Schematic Arrangement; (*b*) Surge-Diagram.

FIG. 13.



Battery—Switch—Inductance—Conduit—Resistance—Ground
(*a*) Schematic Arrangement; (*b*) Surge-Diagram.

Inductance and Resistance in Series; Closing and Opening of Circuit. (Fig. 14.)

The resistance is represented by the straight line of tan ρ slope drawn from the 0 ($= i = 0$, $e = 0$) point. The inductance is represented by the tan γ directrix for which

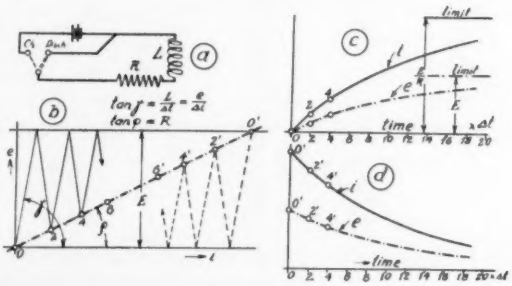
$$\tan \gamma = L/\Delta t = e/\Delta i,$$

where Δt is an arbitrary, small time element. For closing the circuit the successive values of current and voltage will be represented by the 0, 2, 4, 6 points each of which is valid for $2\Delta t$ time-interval. After the elapse of an infinite number of intervals the steady state 0′ ($i = E/R$, $e = E$) is reached. The history of current and voltage for the closing of the circuit is shown in Fig. 14*c*, from which the asymptotic nature of the approach to the steady state can be readily seen.

Assume, that after the attainment of the 0′ steady condition the circuit is broken. The surge diagram of the resulting

transient is represented by the dash-lines giving the points $0'$, $2'$, $4'$, $6'$ $\cdots$, from which the history-diagram of current and voltage, Fig. 14*d*, can be constructed. These again show the asymptotic nature of the approach to the final steady state, 0 ($i = 0$, $e = 0$).

FIG. 14.



Inductance-Resistance; Charging and Discharging. (*a*) General Arrangement; (*b*) Surge-Diagram; full line for charging; dash line for discharging; (*c*) History of Current and Voltage During Charging; and (*d*) During Discharging.

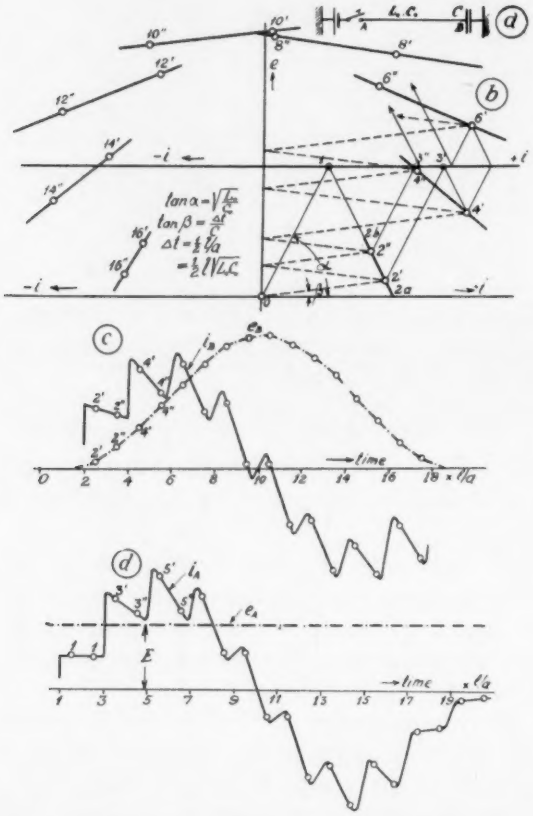
Battery—Switch—Conduit—Capacitance—Ground. (Fig. 15.)

As explained in a previous paragraph the capacitance is represented in the surge-diagram by straight lines having the slope $\tan \beta$ for which $\tan \beta = \dfrac{\Delta e}{i} = \dfrac{\Delta t}{C}$.

The time-element Δt is arbitrary; the smaller it is chosen the more exact the result—and the more laborious the procedure. As explained in the case of inductance, also here it is advantageous to choose Δt equal to T or a submultiple thereof. In this example $\Delta t = T/2 = l/2a$ will be chosen, from which the value of the directrix $\tan \beta$ can be determined and entered into the surge-diagram. When the switch is closed at the beginning of time-reckoning there will start at the switch-end of the conduit a disturbance the coordinates of which are defined by point 1. On reaching the capacitance at the other end of the conduit the voltage is momentarily reduced to zero, and the current doubled according to point $2a$, but after the elapse of Δt time the voltage will increase to a finite value and the current reduced, according to point $2'$ found at the intersection of the $\tan \beta$ directrix drawn from

the point 0 (representing the increase of voltage as a function of influx current), and the tan α directrix drawn from the point 1 (representing the change of current and voltage in the conduit). The increase of voltage and decrease of current

FIG. 15.



Battery—Switch—Conduit—Capacitance—Ground. (*a*) Schematic Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at the Capacitance-End; and (*d*) at the Battery-End of Conduit.

continues; after the elapse of $3\Delta t$ time the state of power $2''$, and after the elapse of $4\Delta t = 2l/a$ interval the state of power $2b$ is attained. Although the current and voltage are continually changing, for the sake of graphical analysis we assume that during the first $2\Delta t$ interval the state $2'$ and during

the second $2\Delta t$ interval the state $2''$ is valid and these waves are propagated toward the battery along the conduit. On reaching the battery the voltage E is re-established with a corresponding increase of current as defined by points $3'$ and $3''$ each of which is assumed to be valid for $2\Delta t$ intervals. These latter waves travel again towards the capacitance-end of the conduit where the states $4'$ and $4''$ are found at the intersection of the tan β and tan α directrices in the manner shown. The procedure is then continued as it is clearly shown in Fig. 15*b*. In this manner for each interval l/a two points are obtained defining the mean current and mean voltage in the first and second half of the interval. From these the sterograms could be constructed but they are omitted here and only the history of current and voltage for the switch-end, Fig. 15*d*, and for the capacitance-end, Fig. 15*c*, of the conduit are shown. It is emphasized that these values are approximative because for full accuracy the Δt interval should be chosen zero while the graphical construction necessitates a finite value for it. For greater accuracy the Δt should be chosen 1/3, 1/4, or lesser submultiple of l/a. If, however, the fluctuations within one l/a interval are of no particular interest, and only the trend of mean values is sought then it is practicable to choose $\Delta t = l/a$ with a resulting saving in labor. This simplified procedure is shown for the circuit:

Battery—Switch—Resistance—Conduit—Capacitance—Ground. (Fig. 16.)

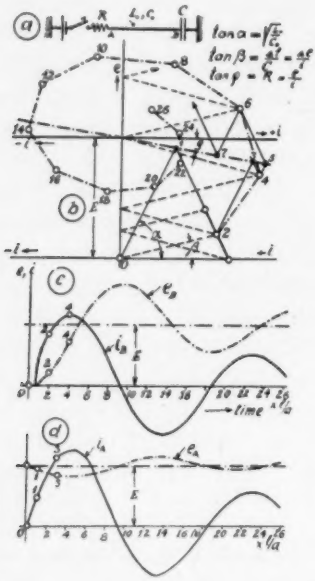
The construction of the surge-diagram is clearly shown and does not require further explanation. At the resistance-end of the conduit the voltage oscillation is little; at the capacitance-end it is greater. The oscillations of the current at the two ends does not show a great difference. The steady condition with $e = E$ and $i = 0$ values is approached in an infinite number of intervals, with ever-decreasing amplitudes of oscillations of current and voltage.

Battery—Variable Resistance—Conduit—Capacitance—Ground (Fig. 16a).

This case is a modification of the one treated above, the resistance being assumed as decreasing from an initial R_0 value to a final R value during a $4\,l/a$ period of time. For

the o to $2\,l/a$ interval the resistance is R_1, for the $2\,l/a$ to $4\,l/a$ interval the resistance is R_2, and from the $4\,l/a$ time onward the resistance is R. The construction is readily understandable from the diagram. It can be visualized that the amplitude of oscillations of current and voltage—after the resistance attained its final R value—will be less and less,

FIG. 16.



Battery—Switch—Resistance—Conduit—Capacitance—Ground. (*a*) Schematic Arrangement; (*b*) Surge Diagram; (*c*) History of Current and Voltage at the Resistance-End; and (*d*) at Capacitance-End of Conduit.

the more slowly, in terms of the l/a periods, the starting resistance is decreased from R_0 to R.

Capacitance and Resistance in Series; Charging and Discharging. (Fig. 17.)

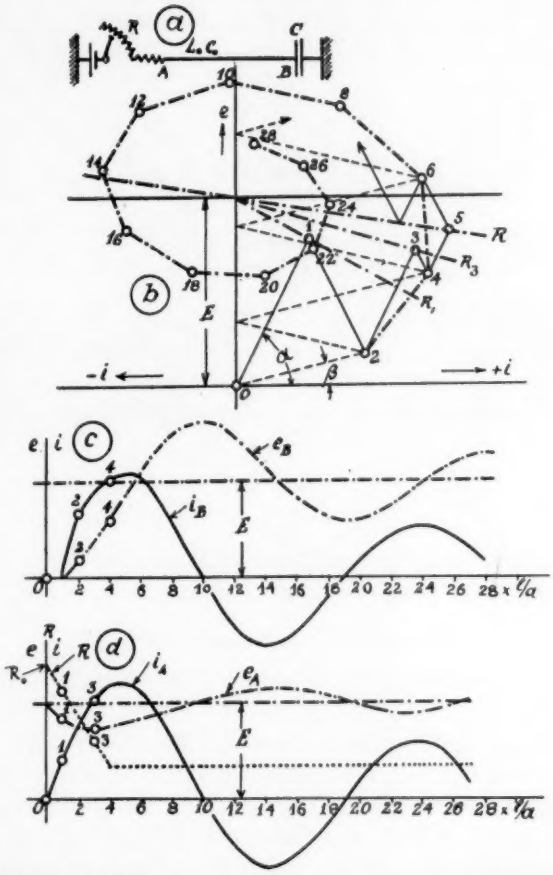
The resistance is represented by the straight line of tan β slope drawn from the o′ ($i = 0$, $e = E$) point. The capacitance is represented by the tan β directrix for which

$$\tan \beta = \Delta t/C = \Delta e/i,$$

where Δt is an arbitrary, small, time-element. For closing

the circuit the successive values of current and voltage will be represented by the 1, 3, 5, 7, ··· points, each of which is valid for $2\Delta t$ interval. After the elapse of an infinite number

FIG. 16*a*.

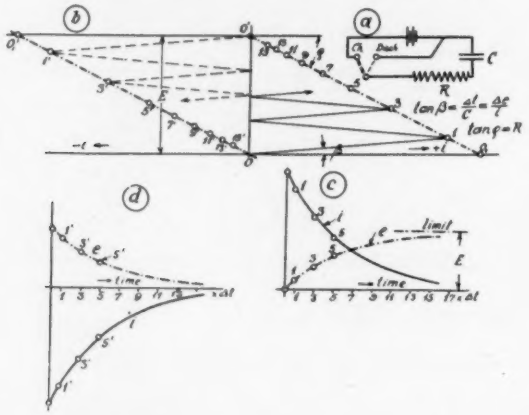


Battery—Variable Resistance—Conduit—Capacitance—Ground. (*a*) Schematic Arrangement; (*b*) Surge Diagram; (*c*) History of Current and Voltage at the Resistance-End and (*d*) at Capacitance-End of Conduit.

of intervals the steady state o′ ($i = 0$, $e = E$) is reached. The history of current and voltage for the charging is shown in Fig. 17*c*, from which the asymptotic nature of the approach to the steady state can be visualized.

Assume that after the attainment of the o′ steady state the switch is thrown over to the discharge position. The surge-diagram of the resulting transient is represented by the dash-lines giving the points 1′, 3′, 5′, 7′, ⋯, from which the history-diagram of current and voltage, Fig. 17*d* can be constructed. These again show the asymptotic nature of the approach to the final steady state o ($i = 0$, $e = 0$).

FIG. 17.



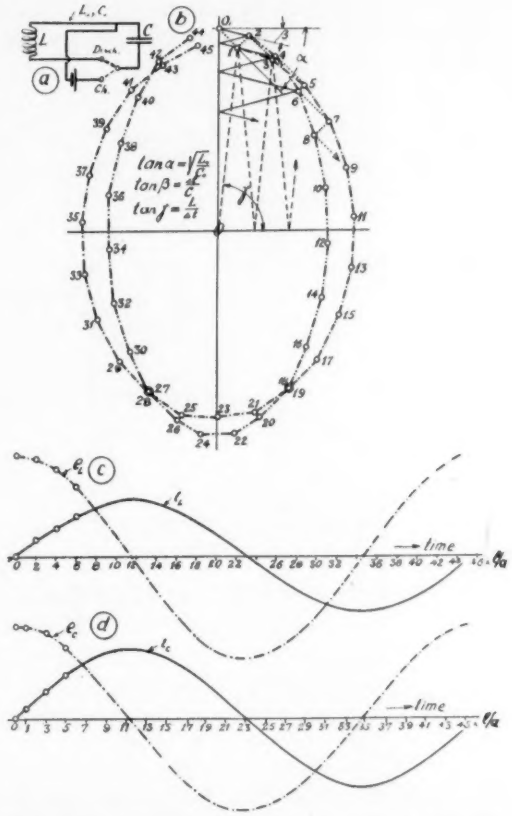
Capacitance—Resistance; Charging and Discharging. (*a*) General Arrangement; (*b*) Surge-Diagram; full line for charging; dash line for discharging; (*c*) History of Current and Voltage During Charging; and (*d*) During Discharging.

Inductance—Conduit—Capacitance. (Fig. 18.)

It is assumed that by means of a two-way switch first the capacitance is charged to the voltage E, and then, by throwing the switch over, the inductance and a long conduit are switched into the circuit. The resulting transient consists in the interchange of the electrostatic energy into electromagnetic energy, and vice-versa, within the three energy-storing elements, (1) the inductance L storing electromagnetic energy, (2) the capacitance C storing electrostatic energy, and (3) the conduit having a distributed inductivity L_0 and distributed capacity C_0 capable of storing both kinds of energy. In the surge-diagram the inductance is represented by the $\tan \gamma = L/\Delta t$ directrix, the capacitance by the $\tan \beta = \Delta t/C$ directrix, and the conduit by the $\tan \alpha = \sqrt{L_0/C_0}$ directrix, where $\Delta t = l/a = l\sqrt{L_0 C_0}$.

At the beginning of time-reckoning the capacitance has the potential E which is impressed on the conduit. On reaching the inductance the voltage will decrease and the current increase; the mean value of the current and voltage

FIG. 18.

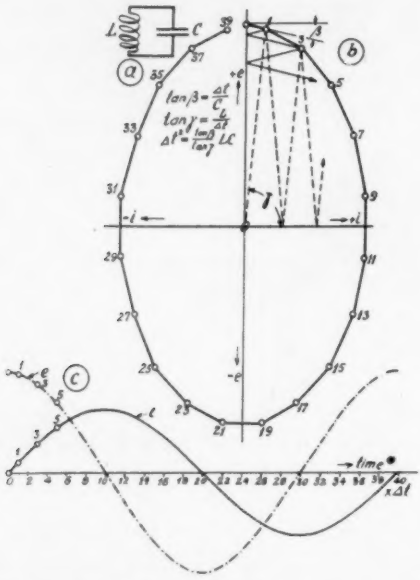


Inductance—Conduit—Capacitance; Discharge. (*a*) General Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at the Terminals of Inductance; and (*d*) of Capacitance.

at the inductance during the first two Δt intervals will be represented by the point 1 located at the intersection of the $0 - 1$ tan γ directrix drawn from the $i = 0, e = 0$ point, with the tan α directrix $0_1 - 1$ drawn from the $1 = 0, e = E$ point. During the second and third intervals the mean value

of the current and voltage at the capacitance will be represented by point 2 being the intersection of the tan α directrix $1 - 2$ with the tan β directrix $0_1 - 2$. The successive points 3, 4, 5, $\cdots$ 45 are obtainable in a similar manner, the even-numbered points 0–2–4–6–$\cdots$44 defining the mean state of power at the terminals of the capacitance, and the odd-numbered points 1–3–5–7–$\cdots$45 defining the mean state of power at the terminals of the inductance, each state being

FIG. 19.



Inductance—Capacitance; Discharge. (*a*) General Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage.

valid for the duration $2\Delta t$. It will be noted that the zig-zagging tan α directrices connect the even-numbered points with the neighboring odd-numbered points, just as in the circuit the conduit connects the capacitance with the inductance. It is seen that each of the two sets of points lie on ellipse-like figures. Constructing from these points the history of current and voltage for the terminals of the inductance and capacitance the Fig. 18*c* and *d* are obtained showing the harmonic character of these changes. In the

absence of energy-dissipating elements these oscillations would continue indefinitely at undiminished magnitude.

Capacitance—Inductance. (Fig. 19.)

If the length (and therefore the inductivity and capacity) of the conduit is negligible then the same values of current and voltage will prevail at the terminals of the capacitance and inductance, and the surge-diagram shown on Fig. 19*b* will result. This is an ellipse-like figure indicating that the current and voltage change in a harmonic manner, the maximum of current corresponding to the zero value of voltage and vice-versa. By diminishing the Δt arbitrary interval a true ellipse would be approximated more and more as the following analysis shows:

For two neighboring points, say 3 and 5 it can be written

$$e_3 - e_5 = -\Delta e = (i_5 + i_3) \tan \beta = (i_5 + i_3) \frac{\Delta t}{C},$$

$$i_3 - i_5 = \Delta i = (e_5 + e_1) \frac{1}{\tan \gamma} = (e_5 + e_3) \frac{\Delta t}{L},$$

whence

$$-\frac{\Delta e}{\Delta i} = \frac{i_5 + i_3}{e_5 + e_3} \frac{L}{C}.$$

Making the transition from the finite Δt to the infinitely small dt, and from the difference-quotient $\Delta e/\Delta i$ to the differential quotient de/di it can be written:

$$-\frac{de}{di} = \frac{2i}{2e} \frac{L}{C} = \frac{i}{e} \frac{L}{C},$$

whence

$$Lidi = -Cede,$$

and integrating we obtain

$$Li^2 + Ce^2 = CE^2,$$

the equation of an ellipse. The horizontal half-diameter represents the maximum value of current, the vertical that of the voltage:

$$i_{max} = E\sqrt{\frac{C}{L}}, \qquad e_{max} = E.$$

Substituting,

$$e = L\,di/dt = LE\sqrt{C/L}\;\omega \sin \omega t;$$
$$i = E\sqrt{C/L}\cos \omega t; \qquad di/dt = E\sqrt{C/L}\;\omega \sin \omega t,$$

it can be written

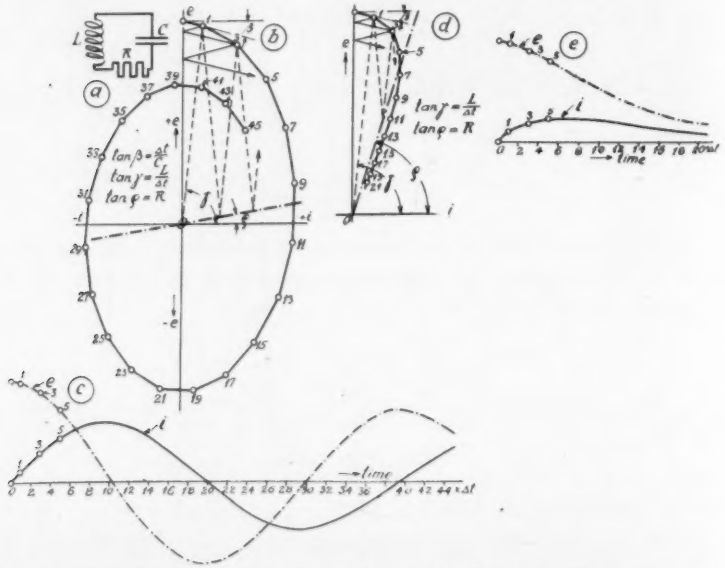
$$Lc\omega^2 \sin^2 \omega t + cos^2\, \omega t - 1 = 0,$$

whence the angular velocity and the period of oscillation can be obtained

$$\omega = \frac{1}{\sqrt{CL}}; \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{CL}.$$

These results are in agreement with those obtained from the algebraic analysis.

FIG. 20.



Inductance—Resistance—Capacitance; Discharge. (*a*) General Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage; (*d*) Surge-Diagram for Large Resistance, Aperiodic Damping; (*e*) History of Current and Voltage for (*d*).

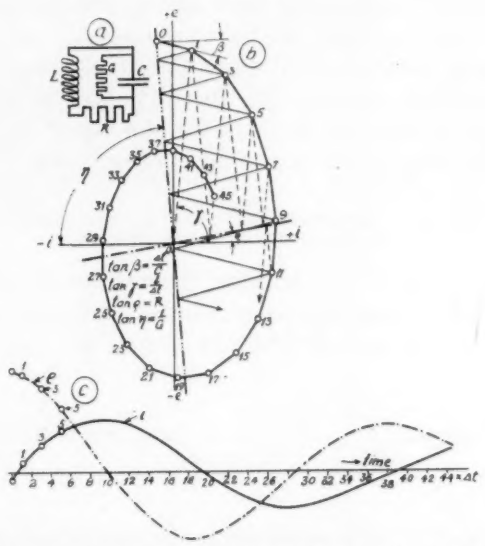
Capacitance—Inductance—Resistance. (Fig. 20.)

If to the circuit of Fig. 19 a resistance is added part of the voltage will be used up by the resistance at each surge of

wave. The construction of the surge-diagram is shown clearly in Fig. 20*b*. If the resistance is small the period of oscillation is not affected appreciably, but the amplitude of both the current and voltage will diminish in an apparently geometrical progression, as it is shown on the history-curve Fig. 20*c*. This is the case of damped oscillation with periodic damping.

If the resistance surpasses a certain value the damping will be aperiodic, i.e. the state of power will approach its final value ($i = 0$, $e = 0$) without oscillation. This case is illustrated in Fig. 20*d* (surge-diagram) and 20*e* (history diagram).

FIG. 21.



Inductance—Resistance—Capacitance—Leakance; Discharge. (*a*) General Arrangement; (*b*) Surge-Diagram; (*c*) History of Current and Voltage at the Terminals of the Inductance.

Capacitance—Inductance—Resistance—Leakance. (Fig. 21.)

Except for the presence of the leakance this circuit is identical to that of Fig. 20. The leakance acts as a shunt between the terminals of capacitance using up a portion of the current existing at the terminals of the capacitance. The leakance is represented by the straight line

$$\tan \eta = \frac{\mathrm{I}}{G}; \qquad Ge = i_G.$$

The transient will be a damped oscillation of current and voltage approaching the final state ($i = 0, e = 0$) more rapidly than in the former case.

CONCLUSION.

The foregoing examples illustrate the range of usefulness of the graphical analysis presented here. For many of the problems treated here the solutions have been already known from previous, algebraic, analysis. The agreement of the results obtained by the two methods is, so to speak, an experimental proof of the correctness of the graphical method. The latter is, however, far simpler to use, and yields a more comprehensive and clear picture of the phenomenon.

The method's advantages go far beyond those of didactic utility. It appears probable that the method can be extended to complicated problems not amenable to algebraic treatment. In its simplicity, clarity and fertility for the dynamics of elastic systems this method can be compared to the graphic statics in the field of statics of rigid structures.

APPENDIX. COMPARISON BETWEEN ALGEBRAICALLY AND GRAPHICALLY OBTAINED RESULTS.

It is of interest to compare the results of the graphical procedure with that of the algebraic analysis for a circuit containing resistance and inductance (Fig. 14). It can be written

for the closure: $E = Ri + L\,di/dt,$ and for the opening: $0 = Ri + L\,di/dt.$

Substituting $E = i_0R$, separating the variables and integrating

$$\int_0^i \frac{di}{i_0 - i} = \frac{R}{L}\int_0^T dt \quad \text{and} \quad \int_{i_0}^i \frac{di}{i} = \frac{R}{L}\int_0^T dt,$$

we obtain: $i = i_0(1 - \epsilon^{-RT/L})$ for the closure and $i = i_0\epsilon^{-RT/L}$ for the opening, giving the value of the current after the elapse of T time after the closure, respectively opening of the circuit ($\epsilon = 2.71828\cdots$).

In the graphical construction we assume that the state

(i.e., current and voltage) is constant during a fraction Δt of the T duration, for which:

$$\Delta t = \frac{L}{\tan \gamma}; \qquad T = \sum_{i=0}^{2n} (\Delta t)_{2i}; \qquad (\Delta t)_2 = \frac{T}{2};$$

$$(\Delta t)_4 = \frac{T}{4}; \qquad \cdots \qquad (\Delta t)_{2n} = \frac{T}{2n}.$$

For a succession of $2\Delta t$ intervals it can be written (Fig. 14), from similar triangles:

$$\frac{i_2}{i_0} = \frac{i_4 - i_2}{i_0 - i_2} = \frac{i_6 - i_4}{i_0 - i_4} = \frac{i_{2n} - i_{2n-2}}{i_0 - i_{2n-2}} = A,$$

where

$$A = \frac{i_2}{i_0} = \frac{E}{\tan \gamma} + \frac{E - i_2 R}{\tan \gamma} = \frac{R}{\tan \gamma}\left(\frac{2i_0 - i_2}{i_0}\right)$$

$$= \frac{R\Delta t}{L}(2 - A),$$

whence

$$A = \frac{1}{\frac{1}{2} + \frac{L}{2R\Delta t}}.$$

For:

$$(\Delta t)_2 = \frac{T}{2}, \qquad (A)_2 = \frac{1}{\frac{1}{2} + \frac{L}{RT}}, \qquad \frac{i_2}{i_0} = 1 - (1 - A_2),$$

$$(\Delta t)_4 = \frac{T}{4}, \qquad (A)_4 = \frac{1}{\frac{1}{2} + \frac{2L}{RT}}, \qquad \frac{i_4}{i_0} = 1 - (1 - A_4)^2,$$

$$\cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots \qquad \cdots$$

$$(\Delta t)_{2n} = \frac{T}{2n}, \qquad (A)_{2n} = \frac{1}{\frac{1}{2} + \frac{nL}{RT}}, \qquad \frac{i_{2n}}{i_0} = 1 - (1 - A_{2n})^n.$$

For infinitely small Δt intervals the last expression will approach the limiting value:

$$\lim_{n=\infty} \frac{i_{2n}}{i_0} = 1 - \left(1 - \frac{1}{\frac{1}{2} + \frac{nL}{RT}}\right)^n = 1 - \left(1 + \frac{1}{\frac{nL}{RT}}\right)^{-\frac{nL}{RT} \cdot \frac{RT}{L}}$$

$$= 1 - \epsilon^{-\frac{RT}{L}} \quad \text{q. e. d.,}$$

which agrees with the algebraically derived value.

In a similar manner the result obtained algebraically for the breaking of the circuit could be derived from the graphical construction, but this will be omitted here.

The value of current and voltage for the charging and discharging of a capacitance through a resistance (Fig. 17) can be obtained by a similar calculation.

For charging: and for discharging:

$$i = C\,de/dt = \frac{E - e}{R}, \qquad -\frac{e}{R} = C\,de/dt.$$

Separating the variables and integrating:

$$\int_0^e \frac{de}{E - e} = \frac{1}{RC}\int_0^T dt \quad \text{and} \quad \int_E^e \frac{de}{e} = -\frac{1}{RC}\int_0^T dt,$$

whence $e = E(1 - \epsilon^{-T/RC})$ for the charging and $e = E_\epsilon^{-T/RC}$ for the discharging of the circuit.

The same result can be derived from the graphical construction, Fig. 17, in a manner, and through a reasoning, similar to that shown for the Inductance–Resistance circuit (Fig. 14). This will be omitted here as the reader can perform it on the previously shown pattern.

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Scientific Knowledge of the Pearl.—In a recent research report by A. E. ALEXANDER of Mellon Institute the attempts made by man to simulate the natural pearl are discussed. Despite human ingenuity and the wonders of science, no artificial or "cultured" pearl can be produced which can compare with that bit of loveliness from the Persian Gulf which is the handiwork of nature. And although cultured and imitation pearls are available, they bear a similar relationship to the natural pearl as gold-filled articles do to solid gold jewelry or silver plate to sterling. Definite scientific tests have been devised so that cultured pearls can be differentiated from natural pearls. One of the simplest methods of distinguishing between these two kinds of pearls is the accurate determination of their specific gravity. By this means it is seen that cultured pearls have an average specific gravity of 2.760, while the natural jewels have an average density of 2.685. The method however, has its limitations. For the past 15 years the X-ray has been used to differentiate natural from cultured pearls. If the gem is natural, a uniform scattering of the X-rays takes place when they impinge on the specimen, producing a six or twelve-fold "spot" pattern. This characteristic pattern is caused by the regular concentric arrangement of the aragonite in a natural salt water pearl. A cultured pearl similarly tested, on the other hand, will usually produce a maltese-cross pattern, or modification thereof; this appearance is attributable to the peculiar parallel mineral arrangement found in all mother-of-pearl substance, the core of a cultured pearl being composed mainly of this material. Another ingenious method used to differentiate cultured pearls from the natural gems employs the "Endoscope," invented by Chilowsky and Perrin in 1926. By this apparatus, light is found to be totally reflected in the interior of a natural pearl, producing a characteristic flash, which does not occur when a cultured pearl is similarly tested by this method. Knowledge of the fact that all mineral matter is either paramagnetic or diamagnetic led to the invention of still another type of test by Richard Nacken in 1929. Differentiation by this method is possible because cultured pearls react differently in a magnetic field than do those of natural origin.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

PROPOSED STANDARD SIZES OF CANS FOR FRUITS AND VEGETABLES.

There has been submitted to the canners, can manufacturers, distributors, consumers, and others interested, a proposed revision of Simplified Practice Recommendation R155–37, "Cans for Fruits and Vegetables."

This simplification project is unique in that the revision of R155–37 was drawn up in terms intended to conform with the broad intent of proposed national legislation first set forth in a Bill (H.R. 6964) introduced in the House of Representatives in 1937 by Honorable Harry Sauthoff, "to fix standards of dimension and capacity for metal containers for canned fruits, vegetables and canned milk in order to prevent fraud and deception. . . ."

In June 1937, following an address before the National Conference on Weights and Measures by Mr. Sauthoff, this Bill, which was designed to effect a mandatory standardization of can sizes, was endorsed by formal resolution of that Conference. In June 1939 the Conference also adopted a report of its special committee on package standardization which recommended that "Federal legislation be initiated standardizing the quantities of all commodities sold in packages or containers of any kind," and including certain detailed recommendations providing for standardization in fixed sizes based on liquid measure, and on avoirdupois weight, and requiring self-differentiation between sizes.

On March 24, 1938 following formal hearing on the Bill 6964 by the Committee on Coinage, Weights and Measures of the House of Representatives to which it had been submitted, Honorable Andrew L. Somers, Chairman of that Committee suggested to the National Canners Association that the canning industry should secure for its own information, and for the information of the Committee, data showing not only

* Communicated by the Director.

the number of can sizes used for each of the industry's products, but also the amount of the respective products canned in each of the sizes now used. In his communication to the Association, the Chairman said:

"Because the industry has been working with the National Bureau of Standards in a can size simplification program, the necessary information can best be collected, I believe, in collaboration with that Bureau. If this Committee can be of any assistance in the collection of the desired information we shall be glad to give it. As I stated at the close of the hearing, I am keeping these hearings open, and the Committee will keep the matter under advisement. We shall be glad to follow what is being done by the industry in its work with the Bureau of Standards."

When the survey was completed in May 1939, the results thereof were brought to the attention of Chairman Somers of the House Committee on Coinage, Weights and Measures, to which had been referred Bill H. R. 4402 of Honorable Andrew L. Somers, representing a virtual reintroduction of Bill H. R. 6964. On July 1, 1939, after consultation with Chairman Somers, as to the exact wording of the communication to "All concerned with the Manufacture, Distribution and Use of Cans for Fruits and Vegetables" the list of recommended sizes of cans was submitted for formal approval by those interested therein.

The revised recommendations were drafted by the Container Simplification Committee of the National Canners Association in recognition of the need of preventing consumer confusion and deception. In designating the cans recommended for use the controlling factor was set forth as the volume of net contents or volume fill.

Recommended by canners for adoption by the industry as standards in packing fruits and vegetables are 44 sizes of cans of specified dimensions. Of these cans, 23 are of special dimensions recommended for use exclusively with individual commodities, namely, asparagus, baby food, corn-on-cob, cranberries, mushrooms, olives, pimientos, pineapples, and tomato sauce. The other 21 cans are recommended for use with from 2 to 56 different designated commodities. Certain of the can sizes are recommended for use with each

of the 33 classifications of vegetables and 24 classifications of fruits, there being never less than 2 nor more than 10 cans recommended for any one classification.

It is understood that the use of the standard sizes of cans for the respective commodities listed in the revised recommendation, if and when approved, will be made effective during the current season, except in instances of outstanding sales-contracts for any of these commodities requiring can sizes not included in the recommended list, and except in cases where non-recommended sizes of cans are already in stock. It is expected that the use of the standard sizes of cans will become fully effective for the 1940 canning season.

Beginning July 1, 1940 the revised Food and Drug Act provides that the label on each can shall state the volume of the content. This will help consumers to determine the actual volume-content of the various cans of food offered for sale after that date.

When accepted and approved by can manufacturers, canners, distributors of canned products, and consumers for promulgation by the Department of Commerce, the revised list of can sizes will set forth concretely the industry's desire for standardization on a voluntary basis. Of special interest to the House Committee on Coinage, Weights and Measures will be the observed adherence to the revised Simplified Practice Recommendation.

Copies of the proposed revision of R155-37 may be obtained by addressing a request to the Division of Simplified Practice, National Bureau of Standards, Washington, D. C.

NINTH PROGRESS REPORT ON INDUSTRIAL USES FOR SILVER.

According to the Ninth Progress Report of the American Silver Producers' Research Project, work, which has been financed for the third and final year, will be chiefly concerned with commercial application of what is already known regarding silver. Previous reports announced that the best chance for silver lies in a wider appreciation of its properties as a noble metal, and no developments thus far have altered this conclusion.

Since the project is conducted in close coöperation with representative commercial companies, the present policy is to transfer problems to industry as soon as a reasonable surety of return can be shown.

There seems to be no reason to lessen the high esteem in which silver is held as a perfectly safe material in contact with foods. As a matter of fact, it is hoped that some systematic study of silver's place in medicine will be undertaken by those competent in this field.

Many methods for applying silver coatings to containers are now being investigated. Several firms are working on the welding of fine silver sheet to a base metal backing and as yet no critical obstacles seem to have been encountered. The chief concern is to discover suitable fields of application and to refer them to the appropriate manufacturer.

Silver-clad base metals made by rolling duplex ingots, are already in limited use and it is gratifying to note that as soon as the commercial demand grows, a place will be found for them in industry. Studies show that the bond remains perfect both mechanically and thermally after great deformation.

Spraying introduces difficulties in the control of coating. Manufacturers of spraying apparatus have been given all information on the subject, and left to capitalize on the advantage of being able to apply a coating in inaccessible places.

It has been found that hot dipping, curiously enough, does not possess any material advantages and, therefore, other methods can be used more advantageously.

In the field of electroplating, steady progress can be reported in the production of thin non-porous coatings by what may be called standard plating technique but with special attention to cleanliness and the conditioning of the cathode surface. The pilot plating plant at the Bureau has been duplicated, and this second plant is available for experimental work other than pilot production.

Brush electroplating seems to possess some advantages, so far as character of coating is concerned, but introduces difficulties in mass production.

A systematic study of the possibilities of plating with a jet to circulate the electrolyte has been started. Although

it is hoped that this method may solve the problem of porosity of very thin coatings, no results are as yet available.

Chemical reduction together with vaporizing and sputtering represent methods for obtaining exceedingly thin coatings (say two millionths of an inch) presumably with some pores. This method has proved satisfactory in some cases, as for vacuum flasks.

Vaporization is being studied on the basis of preliminary tests made at Massachusetts Institute of Technology. This method, which is being tried in test cans, is simple and rapid and apparently adherent coatings are readily made.

Cathodic sputtering has been dropped on account of the long time required to get a coating of appreciable thickness and the apparent lack of any advantages over other methods for our purpose.

In another field, silver contact brushes and rings are being studied according to definite schedule at Rensselaer Polytechnic Institute. The cost of manufacturing brushes is relatively high enough to reduce markedly the price spread on the metal content of silver as compared with copper. The advantages of the silver brushes are the reduction in heat generated and the better equalization of current between brushes in parallel.

Additional work in the field of cold bonding has been done at Lehigh University to find a way of joining silver surfaces without use of brazing material. Heating the surfaces to redness within a reasonable time before placing them in contact facilitates the bond. Some practical application may be found for such a method.

Much has been accomplished in the field of fungicides since the last report. It is a well established fact that ionized silver is toxic to fungal and bacterial pathogens. The spray concentrations required are in most cases too dilute to damage foliage and the costs are easily competitive with bordeaux mixture and the mercurials. Apparently the addition of arsenate of lead as an insecticide creates no interference. At present, practical trials of the work are under way in New York, Maryland, Washington, D. C., Virginia, South Carolina, Georgia, and Florida.

Alloys have given the least return for the work under-

taken of any of the fields so far explored. No justification has been found for adding small amounts of silver to some largely used material such as steel or brass. Neither has any large-scale use been found for some silver-rich alloy, and although some attention has been given to reported magnetic properties of some alloys, no conclusion has thus far been reached. Silver alloy coatings protected by selective anodic oxidation are now being investigated. Excellent welds of silver plated steel have recently been made at Rensselaer Polytechnic Institute by controlled resistance methods. The Bell Telephone Laboratories have developed some silver-rich tin alloys for contacts, and silver-lead bearings are apparently receiving active attention in the airplane field.

In silver-poor alloys, the physical properties of a large assortment of ternary alloys are being studied, and it is hoped that electrical conductivity measurements may show that one or more of them is of importance. It is gratifying to note that the Dow Chemical Company has announced the development of a new wrought alloy containing 5.5 percent silver with some aluminum, zinc, and manganese.

The tin-silver alloys are receiving increasing attention, and apparently the field for a hardened tin is wider than at first imagined. Several steel companies are experimenting with additions of silver to the tinning pots. It seems probable that the International Tin Research and Development Council will place tin-silver on their active list. Tin foil is much hardened by the addition of 3.5 per cent. of silver and a thinner bottle cap lining could be used for mineral waters, etc. Interest is being shown by hospitals and laboratories in silver-bearing tin sheet and pipe for handling sterile or distilled water.

The addition of 0.01 per cent. silver to lead storage battery plates lengthens the life of positive plates, but while it does not help the negative plates, it would be added to both, for manufacturing reasons.

The chief interest in the controversial subject of iron-silver alloys lies in the now published patents covering the campaign, undertaken under the auspices of the Chemical Foundation at the Massachusetts Institute of Technology, for the control of corrosion pitting of stainless steel in salt

water. An "effective" addition of several tenths of a per cent. of silver to "18 and 8" stainless greatly reduces pitting. A general use of this formula would create a demand for a considerable amount of silver and extend its use very appreciably.

STRUCTURAL PROPERTIES OF "PRE-FAB" WALLS, PARTITIONS AND FLOORS.

In its low-cost housing investigation the Bureau tested 33 specimens submitted by the Harnischfeger Corporation. These represented constructions for walls, partitions, and floors marketed under the trade name "Pre-Fab Homes," and consisted of prefabricated panels with welded sheet-steel frames, designed to be erected on masonry foundations and interlocked by special key-and-tapered-pin fastenings.

The wall specimens were subjected to compressive, transverse, concentrated, impact and racking loads; the partition specimens to impact and concentrated loads; and the floor specimens to transverse, concentrated, and impact loads. The transverse, concentrated, and impact loads were applied to both faces of the wall specimens, thus simulating the actual loads applied to the elements of a house.

For each of the loads three like specimens were tested. The deformation under load and the set after the load was removed were measured for uniform increments of load up to the maximum, except for concentrated loads, for which the set only was determined. The results, presented graphically and in tables, are given in Building Materials and Structures Report BMS 18, copies of which are obtainable for 10 cents each from the Superintendent of Documents, Government Printing Office, Washington, D. C.

ACRYLIC RESINS AS DENTURE BASE MATERIAL.

Tests on the curing, "shelf life," color stability, strength, hardness and dimensional changes on curing, were made on three acrylic resins (Crystolex, Lucitone, and Vernonite) furnished the dental profession for use as denture base materials. In comparison with hard rubber they have satis-

factory mechanical properties and are superior in color stability. The shelf life of the two resins furnished in a plastic cake was short at room temperature, while the resin (Crystolex) furnished in powder and liquid was satisfactory in this respect. Observations on dentures in service showed that phenol-formaldehyde resins (Aldenol, Duratone, Luxene) warped and were not color-stable, while an acrylic resin (Vernonite) was color-stable and showed less warpage. When all factors are considered, such acrylic resins as were tested appear to be the most satisfactory plastics currently used for denture bases.

INFLUENCE OF NATURAL NON-TANNINS ON DETERIORATION OF LEATHER.

Leathers may be divided into two groups according to the length of time they are expected to give useful service. One group comprises leathers having a useful life of only a few years; for example, those used for shoes, belting, straps, bags, and miscellaneous articles. Possibly 90 per cent. of the leather made comes within this classification. The other group consists of leathers expected to last for a much longer time, such as those used for upholstery and bookbinding. Indeed, the owner of a valuable book bound in leather hopes that the binding will endure indefinitely. It is generally recognized that one cause of the failure of leather bindings is the destructive action of sulfur gases in the surrounding atmosphere. Methods for protecting leather in service so as to increase its life have, therefore, received considerable attention.

It has been demonstrated by Innes, of the British Leather Manufacturers' Research Association in London, that the non-tannins found in vegetable tanning materials act as protective agents. His conclusion was based on laboratory examinations of various old leathers which were in different stages of deterioration, and also on the results of tests on new leathers to which natural non-tannins were added. The behavior of these samples was observed after treatment with sulfuric acid and hydrogen peroxide. Samples containing non-tannins were less affected by the treatment. Frey and Beebe, of the United States Department of Agri-

culture, came to the same conclusion. They exposed leathers in a chamber to the combustion products of illuminating gas.

The effects of acids on leather have been studied at the Bureau for many years and it has been established that new leather will not deteriorate because of initial acidity if the pH value of the leather is above 3. This criterion is considered sufficient for judging this property of the first-mentioned group of leathers. R. C. Bowker and R. B. Hobbs then decided to determine if possible whether the addition of natural non-tannins would have any influence on the effective acidity, as indicated by pH values, at which leather starts to deteriorate. Chestnut and ordinary quebracho-tanned leathers were prepared for this work in the Bureau's experimental tannery. One portion of these leathers was treated with chestnut extract and another portion with ordinary quebracho extract. By these treatments both tannins and non-tannins were added to the leathers. Each group of samples was treated with sulfuric acid to secure a series having pH values ranging from that of the original leather to about 2.3.

The leathers were tested for initial strength, aged for two years, and again tested for strength. The change in strength on aging was taken as a measure of deterioration. The results show that the addition of small amounts of natural non-tannins—about 0.5 to 2.6 per cent.—and the addition of about 6 per cent. of tannins, did not influence the initial acidity at which these leathers start to deteriorate. Serious deterioration started at or near pH 3, as was the case in many previous studies on vegetable-tanned leathers.

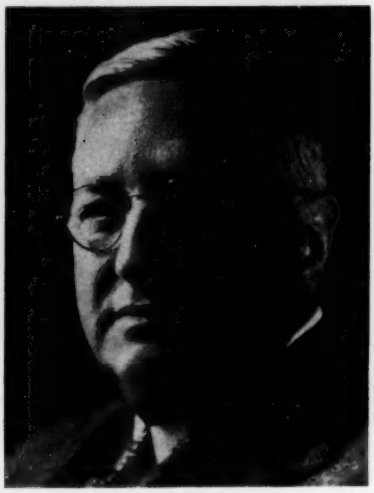
Time and the Growth of Physics.—DR. ARTHUR H. COMPTON, Professor of Physics at the University of Chicago and winner of a Nobel Prize for Physics, discussed this subject recently at New York University in an address which was the eighth annual James Arthur Lecture on Time and Its Mysteries. He stated that time, space, and mass form a triad in terms of which the physicist tries to account for his entire world. Thus the concepts of dynamics, such as motion, speed, force, energy and power, all involve a knowledge of time. But in order that these concepts shall have definite meaning, we must know the observer's situation relative to the events that are to be described. This is an aspect of time that was especially emphasized by Einstein. Within the last year, two important examples have arisen of clocks whose rate of motion is affected by their state of motion relative to the observer. In one of these cases, Dr. H. E. Ives has shown that when the light emitted by a moving atom is studied, its rate of vibration is perceptibly reduced when the atom is moving with an appreciable fraction of speed of light. In Ives' experiment, the moving atom is in a beam of ionic rays produced in the laboratory. The second example of a clock whose rate is altered by its motion is that of the penetrating cosmic rays known as mesotrons. Within the past year it has become evident that these particles are produced in the earth's upper atmosphere by the action of some high energy primary ray which is itself completely stopped in the earth's upper air. The mesotrons then proceed through the air to sea-level, or some of them deep into the earth, moving at very nearly the speed of light. The evidence has now become strong that these mesotrons are ephemeral, acting like radioactive atoms but with an average life of only about a ten-thousandth of a second.

R. H. O.

THE FRANKLIN INSTITUTE.

VERMILYE MEDAL.

"In recognition of outstanding contributions to the science of industrial management," The Franklin Institute will award biennially, or at a longer interval, a medal to one who has distinguished himself in the field of industrial management. Named after its donor, William M. Vermilye, the purpose of the medal is to stimulate, encourage and recognize outstanding contributions in the field of industrial management not only in the United States but in other countries as well.



In the not too distant future the first award of the Vermilye Medal will be made by the Institute through the action of its Board of Managers on the recommendation of a committee of board members and an Advisory Committee, including prominent engineers and industrialists and representatives of the American Society of Mechanical Engineers, the Chamber of Commerce of the United States and the National Association of Manufacturers.

That The Franklin Institute should have this opportunity and responsibility is in keeping with its long and honorable record of public service in the spirit of Benjamin Franklin. Dr. Sydney L. Wright in his recent work, "The Story of The Franklin Institute," makes the following pertinent statement:

"The Franklin Institute is devoted now, as it has been for one hundred and fifteen years, to the increase of useful knowledge, to the encouragement of invention and discovery,

and to the education of the public in the achievements of science and industry. Its very title has always indicated a desire to honor Benjamin Franklin. Recently, due to its position as the center of the Benjamin Franklin Memorial, all its activities are in a sense devoted to his honor, not only as a scientist, but as a man. It may truly be said that the Institute has the homely wisdom of Poor Richard before it and shares his desire to serve humanity."

Thus the award of the Vermilye Medal does not constitute a precedent for The Franklin Institute but rather does it evince the firm conviction of the Institute that formal recognition not only gives honor to him who deserves such distinction but acts as a stimulant and an incentive for others. Down through the years the Institute has used this means of acknowledging contributions made to science and technology. And, by bestowing its medals upon living inventors, engineers and scientists the Institute has demonstrated that a man's worth can and should be evaluated while he has yet the power to perform additional services in the common cause of human advancement. It thus encourages him to do more of like work and inspires others to worthy emulation.

Among the awards made by the Institute upon the recommendation of its Committee on Science and the Arts are: The Franklin Medal, The Elliott Cresson Medal, The Howard N. Potts Medal, The Louis Edward Levy Medal, The George R.

Henderson Medal, The Walton Clark Medal, The John Price Wetherill Medal, The Edward Longstreth Medal and The Frank P. Brown Medal.

The Franklin Medal, "to those whose efforts have contributed to a knowledge of physical science and its applications," has been awarded since 1915 to fifty eminent scientists, engineers and inventors, including Arrhenius, Berliner, Bragg, Edison, Einstein, Fleming, Hubble, Jeans, Jewett, Kettering, Langmuir, Lenard, Marconi, Millikan, Modjeski, Nernst, Planck, Rutherford, Sauveur, Sprague, Elihu Thomson, J. J. Thomson, Wright and Zeeman.

In the long list of recipients of the Elliott Cresson Medal, founded in 1848, may be found not only those who have made "distinguished contributions in the realm of physical sciences," but those who have figured in the industrial development of America. In alphabetical order they are:

Alexander Graham Bell, telephone; Emile Berliner, telephone and sound reproduction; Walther Bauersfeld, optical planetarium; G. H. Clamer, methods of eliminating metals from mixture of metals; Juan de la Cierva, autogyro; Pierre and Marie Curie, radium; Lee DeForest, the audion; Diesel, motor; Henry Ford, automobile; Frederick E. Ives, color photography; Chevalier Jackson, bronchoscope; Edwin H. Land, "polaroid"; T. Lanston, monotype machine; Ernest O. Lawrence, cyclotron; Isaac Newton Lewis, machine gun; Karl P. G. Linde, refrigeration processes; Auguste and Louis Lumiere, color photography; O. Mergenthaler, linotype; Edwin F. Northrup, electric furnace and high temperature investigations; Michael J. Owens, bottle blowing machine; W. C. Roentgen, x-ray; Sir Ernest Rutherford, advancement of knowledge of electrical theory; Albert Sauveur, metallography of iron and steel; Elmer A. Sperry, devices based on the principles of the gyroscope; Frank J. Sprague, multiple unit system of electric traction; Charles P. Steinmetz, application of analytics to electrical engineering; Elihu Thomson, industrial applications of electricity; J. J. Thomson, leading work in physical science; W. V. Turner, air brake design and application; Samuel M. Vauclain, compound locomotive; L. Walkup, air brush; Carl Auer von Welsbach, incandescent mantles; Harvey W. Wiley, leading work in agricultural chemistry; and Orville Wright, work in aviation.

It is interesting to note that among those who received The Elliott Cresson Medal and later were given the highest award of the Institute are: Emile Berliner, Ernest Rutherford, Albert Sauveur, Frank J. Sprague, Elihu Thomson, J. J. Thomson and Orville Wright.

While space does not admit of a listing of some of those who have received the other medals of the Institute it is significant that Modjeski who received The Franklin Medal had previously been given The Howard N. Potts Medal for his paper on the "design of large bridges with special reference to the Quebec Bridge," which appeared in the JOURNAL OF THE FRANKLIN INSTITUTE for September 1913.

These instances and others not mentioned would indicate that one honor begot another and that one triumph was the basis for another success.

In accepting the Vermilye Medal, the officials of The Franklin Institute have based their action on years of accumulative experience in making medal awards in the field of science and the mechanic arts. In the awarding of the new medal they hope and expect that recognition from time to time of outstanding contributions to the science of industrial management will produce equally beneficent results.

The high regard that the Institute holds for the Vermilye Medal and all that it symbolizes and all that it promises as a constructive force, is attested by the caliber of men who have been entrusted with the selection of the first recipient. They are:

Standing Committee on the Vermilye Medal: Mr. Conrad N. Lauer, chairman; Mr. Charles Penrose, Mr. Edward G. Budd, Dr. Henry Butler Allen, ex officio; Advisory Committee on the Vermilye Medal: Mr. William L. Batt, Mr. Charles E. Brinley, Mr. H. R. Martz, Professor Joseph W. Roe, Mr. R. A. Wentworth, representing the American Society of Mechanical Engineers; Mr. John W. O'Leary, Chairman of the Executive Committee of the Chamber of Commerce of the United States; and Mr. Charles W. Hook, representing the National Association of Manufacturers.

The Vermilye Medal itself implies a tribute to the practical idealism of Dr. Franklin, for the obverse side of the medal reveals the likeness of America's great printer, patriot, sage

and scientist as immortalized by James Earle Fraser in the heroic statue which is enshrined in the national Benjamin Franklin Memorial in Philadelphia. The medal, of bronze, was designed by Mr. Gerald K. Geerlings and executed by Mr. Allen Townsend Terrell, sculptor. The reverse bears the following inscription: "Awarded by The Franklin Institute of the State of Pennsylvania in recognition of outstanding contribution in the field of industrial management to . . . ."

That the medal should have been founded by Mr. William Moorhead Vermilye is particularly appropriate in view of his many years of experience in industry and finance during which time by long, hard, every day effort to meet the problems of industrial administration he has built a solid foundation for his present work as Vice-President of the National City Bank of New York.

No story of the Vermilye Medal Award would be complete without a word picture of the man who conceived it. Mr. Vermilye has had a varied business career, rich in those contacts which would be a natural prelude to the inspiration for recognition of the executive who has done most to bring management and labor into closer understanding.

Mr. Vermilye was born in Orange, New Jersey, the son of Daniel Babbitt Vermilye and Mary Cornelia Holmes Vermilye. He is a descendant of Johannes Vermelje, an early Manhattan settler. At the age of seventeen, after leaving Staten Island Academy, a preparatory school, he entered the class of 1901 at the Massachusetts Institute of Technology. Young Vermilye had not been at college two years, when his step-father started a textile business and gave the youth his choice of remaining at school or going into business with him. Mr. Vermilye chose the opportunity of learning the textile trade.

Then followed years in the textile industry, first as clerk, then salesman, later in successively higher executive posts.

It was while operating the Manhasset Manufacturing Company for the National City Bank of New York that Mr. Vermilye had his first opportunity to experiment with certain ideas having to do with employer-labor relations. Success attended his thoughtful efforts, and he was able to carry his manufactory through the troubled year of 1922 without actual loss.

After subsequent reorganization activities with numerous industries, Mr. Vermilye returned to the National City Bank in 1937 as a Vice-President.

The first award of the Vermilye Medal, according to present plans, will be made by The Franklin Institute, in Philadelphia, Tuesday, November 14, 1939. The recipient has not been announced.

LIBRARY NOTES.

The Committee on Library desires to add to the collections of the Institute any technical writings of members who have had occasion to publish such material.

Literary contributions from author-members will be gratefully acknowledged, properly inscribed and noted in the Journal of the Institute.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The cost for a print 9 × 14 inches is forty cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays two until ten o'clock P.M. 590 readers made use of the facilities during the twenty-five days of July.

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NOTES FROM THE BARTOL RESEARCH FOUNDATION.

THE COSMIC-RAY INTENSITY AT HIGH ELEVATIONS IN NORTHERN LATITUDES.*

BY

T. H. JOHNSON.

The airplane flights with an unshielded electroscope made by Bowen, Millikan and Neher [1] up to an elevation corresponding to an atmospheric depth of 4.5 meters of water showed about a twenty per cent. increase in cosmic-ray intensity between the magnetic latitudes 41° N. and 54° N. but from there on up to 63° N. there was no further increase. Their failure to find a dependence upon latitude at these highest latitudes could have been interpreted either as an inability of rays of energy less than the 2.2 billion volt limit corresponding to 54° latitude to penetrate to the depth of 4.5 meters, or as an absence of such low energy rays in the primary cosmic-ray spectrum. If the knee at 40° N. in the curve representing intensity at sea level *vs.* latitude were due to the stopping of the corresponding 6-billion volt rays in ten meters, and if range were proportional to energy, one would have expected the stopping power of the atmosphere at 4.5 meters depth to produce a knee at 50° N., and therefore no variation of intensity above 54° N. could have been expected on either hypothesis. The same conclusion is reached by considering the effect of the soft component at this depth in the theory of Bhabha and Heitler.[2]

In view of the interesting suggestion of Vallarta [3] and Janossy [4] that the magnetic field of the sun might be sufficient

* Reprinted from the *Physical Review*, **54**, 151 (1938).

[1] I. S. Bowen, R. A. Millikan and H. V. Neher, *Phys. Rev.*, **46**, 641 (1934).

[2] H. J. Bhabha and W. Heitler, *Proc. Roy Soc.*, **A159**, 432 (1937). A convenient table for calculation is given by L. W. Nordheim, *Phys. Rev.*, **53**, 694 (1938). The probability of a ray of energy greater than 10^8 ev. appearing at a depth of 4.5 meters when one ray of 2.2×10^9 ev. enters the atmosphere is about 0.05.

[3] M. S. Vallarta, *Nature*, **139**, 839 (1937).

[4] L. Janossy, *Zeits. f. Physik*, **104**, 430 (1937).

to exclude these low energy radiations from the region within the earth's orbit, with the consequence that the cosmic-ray spectrum would possess a low energy limit other than that imposed by the earth's magnetic field, a series of balloon flights were planned and carried out in August and September 1937 in central Minnesota, 56° N., and in Churchill, Manitoba, 69° N. Vertical coincidence counters were used for the cosmic-ray measurements in order to realize shorter path lengths through the atmosphere, and the ultra-high frequency radio technique was employed for transmission of the data.[5] Each coincidence was transmitted as a dot and at regular intervals a series of dashes counted off the number of immersed taps on a mercurial barometer so that the exact times when the balloon passed through definite pressure intervals were recorded. The counters were 6 cm. long by 1.2 cm. in diameter and were placed horizontally one above the other with a 2.5 cm. separation between axes. In this configuration the coincidence rate at sea level was about 2 per minute. The counters were filled with 25 cm. of a one to one mixture of neon and hydrogen and operated at about 1100 volts. The discharges were quenched with 300 megohm resistors and the plateau of uniform counting rate extended over such a wide range of overvoltage that it was possible to use a capacity multiplication of the B voltage for the operation of the counters without trouble from counter voltage variations. A multivibrator type of feedback circuit [6] was used for prolongation of the pulses from the coincidence selecting stage and this could be tripped off with such a small input of power that exceedingly short resolving times were realized; with the radiation intensity equal to that encountered in the stratosphere there was no appreciable alteration of the coincidence rate either from accidentals or from the inefficiency resulting from the high counting rate.

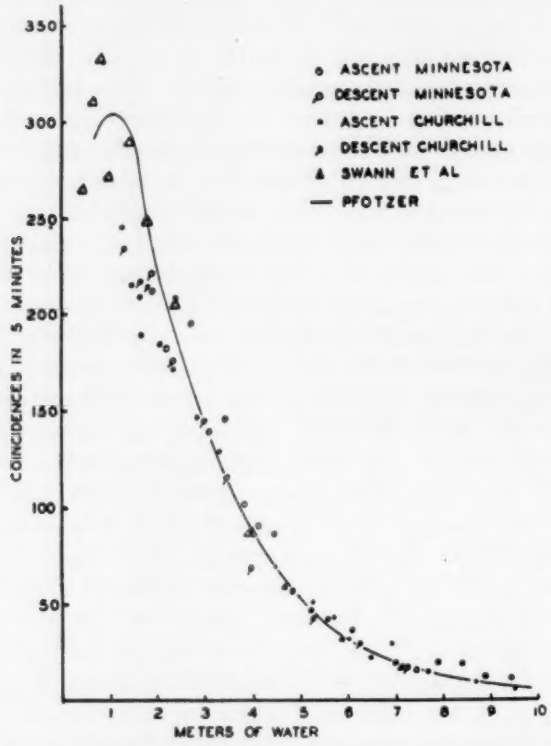
The shaded points in Fig. 1 indicate the numbers of coincidences recorded in five minute intervals on a flight made in Churchill, plotted against the mean pressure during the interval. The open points are the corresponding data for a

[5] For a fuller description of the experimental technique see T. H. Johnson, J. FRANK. INST., **223**, 218 (1937).

[6] T. H. Johnson, *Rev. Sci. Inst.*, **9**, 218 (1938).

flight in Minnesota with exactly similar apparatus. The curve represents the corrected data of G. Pfotzer [7] obtained at the latitude 52° N. and the triangles represent the data of Swann, Locher and Danforth.[8] Both these and Pfotzer's data

FIG. 1.



Cosmic-ray intensities measured with vertical coincidence counters. Results in Churchill mag. lat. 69° N. are compared with similar data at lower latitudes.

were obtained with triple coincidence counters in the vertical direction. They have been adjusted by suitable factors to bring about agreement in the central part of the curve.

The results indicate no increase of intensity with latitude north of Minnesota (56° N.) up to an atmospheric depth of two meters of water. If the sea-level knee were attributable

[7] G. Pfotzer, *Zeits. f. Physik*, **102**, 23 (1936).

[8] W. F. G. Swann, J. FRANK. INST., **224**, 415 (1937).

to absorption of rays of energy less than 6 billion volts then with proportionality between range and energy a knee at two meters depth could be expected at 60° N. where, according to Lemaitre and Vallarta, 1.2-billion volt rays are excluded by the earth's field. Thus it would have been reasonable to expect an increase of intensity at Churchill over that found at Minnesota equal to the contribution of rays in the energy range between 1.2 and 1.5 billion volts. The fact that no such increase was found favors the interpretation that there is very little intensity in the primary spectrum within this range of energies,[9] but in view of the experimental inaccuracies which may be judged by the dispersion of the points about a smooth curve and especially of the uncertainties in present knowledge of the stopping power of air for these low energy rays, it is impossible to insist too strongly upon this interpretation. It is possible that coincidence counter flights to still higher elevations would be able to give a more conclusive answer to this question.

The writer wishes to acknowledge the financial assistance given this work by the Carnegie Institution of Washington, as well as the assistance of various individuals. Mr. John Marshall, Jr., coöperated in making the flights. The work in Minnesota was carried out with the help of the Minnesota Department of Conservation through the courtesy of G. M. Conzet. Mr. J. Patterson of the Canadian Meteorological Service helped in arranging the Churchill flights.

[9] This conclusion is in agreement with the results presented at the Washington Meeting of the American Physical Society by Bowen, Millikan and Neher.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Electrophoresis as an Aid to Preparative Ultracentrifugation.—THOMAS J. DIETZ. (Physico-Chemical Department of the Biochemical Research Foundation of the Franklin Institute, Philadelphia, Pa.) The field of protein chemistry has been extended materially by the development of the analytical ultracentrifuge. This apparatus makes possible the subjection of protein solutions to centrifugal force sufficiently intense to cause the protein in solution to sediment, and permits the observation and recording of this sedimentation under controlled conditions by means of a suitable optical system. In consequence of ultracentrifuge studies, many proteins are now regarded as possessing characteristic sedimentation constants (for details see "The Ultra-Centrifuge and the Study of High Molecular Weight Compounds," by Prof. The Svedberg, *Nature*, **139**: 1051, 1937).

Since a differential sedimentation can result in the separation of proteins of unlike sedimentation constants, ultracentrifuges of the air-driven type have been devised for this purpose (a review of air-driven ultracentrifuges may be obtained from "High Speed Centrifuging," by Dr. J. W. Beams, *Review of Modern Physics*, **10**: 245, 1938). From these ultracentrifuges, which are designed to handle relatively large volumes of material, the supernatant and sediment fractions of the centrifuged specimens can be recovered for chemical or biological assay. The preparative type of ultracentrifuge may be either: (1) a quantity ultracentrifuge which is capable of handling volumes up to 200 cc. at one time, or (2) a continuous separation ultracentrifuge through which fluids to be fractionated may be passed continuously. Ultracentrifuges of the latter type offer opportunities for application in commercial preparative procedures.

Together with the ultracentrifuge analytical method for determining molecular homogeneity and sedimentation constant of a protein, have come electrophoretic methods for

determining electrochemical homogeneity and electrophoretic mobility. Probably the most suitable method for measuring electrophoretic mobility is that devised by Tiselius, the design of which was the result of considerable research (for more complete information, see "A New Apparatus for Electrophoretic Analysis of Colloidal Mixtures," by Arne Tiselius, *Trans. Farad. Soc.*, **33**: 524, 1937). The electrophoresis apparatus of Tiselius enables the obtaining of data on the electrochemical state of any one of several proteins in solution. Electrophoretic analysis of a complex mixture of proteins in solution defines the number and relative concentrations of all electrochemically distinct components present if they are not in too small concentration to be beyond the sensitivity of the optical system of the apparatus. The electrochemical homogeneity and the electrophoretic mobility of each component are also included in the definition, and if mobility measurements are made at different hydrogen ion concentrations, the iso-electric point of each component is determinable by interpolation. Therefore, it is possible to correlate alterations in electrochemical homogeneity with the deleterious effects of chemical or physical treatment, to correlate changes in electrophoretic mobility with chemical combination, and to define the specificity and efficiency of ultracentrifuge fractionation procedures from measurement of relative concentrations of components found in the supernatant and sediment fractions. The usefulness of such data in preparative ultracentrifugation is unquestionable because the results of ultracentrifuge fractionation are often indefinite concerning the identity of proteins in a fraction. Examples are presented in the following paragraphs to show how electrophoretic analysis can be utilized in ultracentrifuge fractionation procedures.

Applications of the preparative ultracentrifuge are limited when fractionation is dependent alone upon native molecular weight differences. The possible ultracentrifuge purification of virus proteins for use as immunizing agents is evident, but at present such procedure is impracticable for technical reasons related to pathogenicity. The use of electrophoresis in conjunction with ultracentrifuge purification of virus proteins has not been investigated extensively. Conse-

quently, the question of the possibility of electrophoretic control of methods for inhibiting pathogenicity preparatory to ultracentrifugation cannot be answered at present. Since, however, native viruses are infective in minute concentrations, i.e. beyond the sensitivity of the optical system of the electrophoresis apparatus, the only apparent possibility for use lies in the control of degradation of the purified product.

The conditions for ultracentrifuge purification of virus protein are ideal in principle in that the readily separable component is the virus protein itself; more general applications of ultracentrifuge purification present difficulties because the protein to be purified is contaminated with undesirable proteins whose sedimentation constants are often of the same order. Therapeutic antisera are examples of this type of system, where the protective antibody is the desired protein contaminated with an excessive amount of inert protein. The molecular weight of the antibody from antipneumococcus horse serum is represented in the literature to be roughly six times that of any of the inert protein components. These molecular weight determinations were taken from ultracentrifuge data, the analyses being made on antiserum obtained from freshly immunized horses and diluted three or more times. In undiluted serum the antibody protein tends to dissociate into components of lower molecular weights. It has also been found that the antibody fraction from the serum of repeatedly immunized horses is only partially of the homogeneous high molecular weight species. A practical ultracentrifuge method for antibody purification is not set up to meet limitations such as are mentioned above. The only apparent alternative is to change the molecular weight of the antibody component so as to insure efficient fractionation.

Experiments conducted in this laboratory have indicated that it is possible to heat-treat antipneumococcus horse serum so as to cause aggregation of some of the serum proteins. Ultracentrifuge tests proved that, under controlled conditions, the aggregation is semi-specifically confined to the antibody fraction of the serum. The possibilities for practical ultracentrifuge fractionation based upon such procedure have not been fully determined, but it appears that the idea of aggregating a specific component for purification purposes is not

untenable. In this connection, it was evident from electrophoretic control experiments that the technique of electrophoretic analysis can be used to guide ultracentrifuge fractionation.

Electrophoretic control of chemical or physical treatment of therapeutic antisera materially aids in defining the condition of the system. For instance, when antipneumococcus horse serum is subjected to mild heating at temperatures ranging from 56° C. to 65° C., certain progressive changes take place in the appearance of the electrophoretic diagram. The protein component with which the antibody activity is associated appears to undergo some minor electrochemical change evidenced by a slight broadening of the boundary;[1] this change accompanies the semi-specific aggregation of the antibody component. Another component, usually referred to as the β component, exhibits a moderate increase in concentration. With increased heat treatment the electrochemical homogeneity of the antibody component is further reduced, as is also its concentration. The β component at the same time exhibits an additional increase in concentration. Heat treatment has not been carried beyond this point because further heating results in gelation of the antiserum.

Similar alterations can be produced by allowing the antiserum, to which a small amount of formalin has been added, to stand over night at room temperature. On the following day the antiserum has an appearance of opalescence similar to that resulting from heat treatment. Electrophoresis of the formolized antiserum indicates changes qualitatively similar to those accompanying heat treatment, but with the formolized antiserum, the alteration can be carried further without causing gelation. A slight increase in the concentration of formalin added causes an aggregation which apparently includes all globulin components because electrophoretic analysis reveals only one globulin boundary in the formolized antiserum, i.e., the aggregated component. This component

[1] The term "boundary" refers to the interface or the refraction gradient which is created between the protein in solution and its solvent at the beginning of the experiment. The behavior of the boundary in an electric field is correlated with electrophoretic characteristics of the protein which constitutes the boundary.

is homogeneous with respect to charge, and has the approximate mobility of the original β component.

If electrophoretic analyses are made on the supernatant and the sediment fractions of ultracentrifuged antipneumococcus horse serum (undiluted), the electrophoretic diagrams of the two fractions appear qualitatively alike. All serum protein components are present in the sediment fraction in slightly greater concentration than in the supernatant fraction. In some specimens of antiserum, the globulin component associated with antibody activity may be relatively increased in the sediment fraction. In general, the similarity of the electrophoretic diagrams of the supernatant and sediment fractions of the ultracentrifuged antiserum indicates that there can be no specific or semi-specific sedimentation of any protein component in the unaltered antiserum.

In the case of heat-treated antipneumococcus serum, the situation is somewhat changed. When a certain protein component is aggregated as a result of heating, the sedimentation constant of this component will differ markedly from those of unaggregated components. Electrophoretic analysis of the supernatant and sediment fractions of the ultracentrifuged heated antipneumococcus horse serum reveals: first, that the component presumably associated with antibody activity is almost entirely absent from the supernatant fraction; second, that a highly aggregated, electrochemically monodisperse component is present in the sediment fraction, its mobility being approximately that of normal β globulin. About seventy-five per cent. of this component is specifically precipitable with homologous polysaccharide; specific precipitability is normally an attribute of the antibody component.

Considering the cases mentioned above, it seems evident that electrophoretic analysis of a protein system, such as antipneumococcus serum, yields data useful to the quantitative control of preparative ultracentrifugation. Such analytical data are particularly helpful when physical or chemical treatment is required for specific fractionation.

A Titrimetric Modification of the Glyoxalase Method for the Estimation of Reduced Glutathione.—E. F. SCHROEDER AND GLADYS E. WOODWARD. (*Journal of Biological Chemistry*,

129: 283, 1939.) Much attention has been given in recent years to the question of the physiological significance of glutathione. This tripeptide, composed of the amino acids glycine, cysteine, and glutamic acid, appears to be almost universally distributed in all types of plant and animal tissues. At present not a great deal is known of its role in vital processes. The cysteine portion of the molecule contains a sulfhydryl (–SH) group which is easily oxidized to the disulfide (S–S) form. The reverse reaction, reduction of S–S to –SH also occurs readily. For this reason it is believed that reduced glutathione (GSH) and oxidized glutathione (GSSG) form an intracellular oxidation-reduction system which functions in cellular respiration. Glutathione also has been found to activate certain enzymes, such as glyoxalase and cathepsin; thus it probably serves *in vivo* as a regulator of certain metabolic processes.

Our lack of precise information in this field is due in large part to inadequacies of the methods in general use for the quantitative estimation of glutathione. These methods usually have been based on the oxidation of GSH to GSSG by means of iodine, ferricyanide, and other oxidizing agents. The results thus obtained are subject to gross inaccuracies because (*a*) external conditions such as temperature and concentration often influence the course of the oxidation, and (*b*) many other tissue constituents are oxidized to a greater or lesser extent by the above oxidizing agents. As a consequence glutathione values for individual tissues, as reported in the literature, may vary by several hundred per cent., depending on the method used. Obviously, little reliance can be placed on conclusions drawn from such data.

In 1935 one of us (Woodward, *J. Biol. Chem.*, **109**: 1, 1935) described a procedure, based on a new principle, for the quantitative estimation of glutathione. Lohmann had found a short time previously that reduced glutathione functions as the activator of the enzyme glyoxalase, which converts methylglyoxal (CH_3-CO-CHO) into lactic acid (CH_3–CHOH–COOH) by introducing a molecule of water. The extraordinary specificity of reduced glutathione in catalyzing this reaction is the basis of the Woodward method. When no glutathione is present, the enzyme exhibits no

activity whatever; in the presence of glutathione the enzyme becomes active and the degree of activity is dependent upon the quantity of the tripeptide in the reaction mixture. No other substance has been found which has any activating power toward glyoxalase; and even slight changes in the structure of the glutathione molecule will destroy its ability to influence glyoxalase action.

In the original method of Woodward, a reaction mixture consisting of methylglyoxal, glyoxalase, sodium bicarbonate, and the glutathione-containing solution being tested, is shaken for 20 minutes at 25° C. in the Warburg manometric apparatus. As lactic acid is formed under the influence of the enzyme, it is neutralized by the bicarbonate with evolution of CO_2. The amount of CO_2 formed is a measure of the rate of glyoxalase action, which in turn is dependent upon the quantity of glutathione present. By reference to a standard curve, obtained by carrying out a series of similar reactions in the presence of known amounts of pure glutathione, the quantity of the latter present in the test solution can be estimated.

Because of its specificity, this method has proved to be of considerable value. However, its use is restricted to laboratories equipped for manometric work. The apparatus is quite expensive and the technique requires considerable experience. In view of these facts, a modification has been developed in which the use of this rather specialized technique is avoided.

The modified procedure is similar, in principle, to the above. However, the rate of glyoxalase activity, instead of being followed manometrically, is determined by titration of the unchanged substrate, methylglyoxal, after completion of the enzyme action. A reaction mixture containing a known amount of methylglyoxal, glyoxalase, phosphate buffer of pH 7.2, and the glutathione-containing solution under examination, is incubated for exactly 30 minutes at 25° C. in a 25 cc. Erlenmeyer flask. At the end of this period, N/50 H_2SO_4 is added to stop the enzyme action, and the unchanged methylglyoxal determined by the iodometric-bisulfite titration procedure of Clift and Cook (*Biochem. J.*, **26**: 1788, 1932). The decrease in methylglyoxal present is dependent

upon the amount of glutathione in the reaction mixture. The latter may then be estimated by reference to a standard curve obtained by carrying out similar reactions with known amounts of pure glutathione.

The iodometric procedure of Clift and Cook, referred to above, has been slightly modified in order to improve the stability of the end-points. In principle, the method is as follows. A 5 cc. aliquot of the methylglyoxal-containing solution is allowed to stand for 15 minutes at room temperature with 1 cc. of molar sodium bisulfite. During this time the methylglyoxal forms an addition compound with the bisulfite (through the carbonyl groups). The excess bisulfite is then removed by addition of strong iodine, starch being used as indicator. After 15 minutes standing at 25° C. the solution is made slightly alkaline, which causes the methylglyoxal-bisulfite complex to dissociate. The bisulfite thus liberated is finally titrated with 0.005 N iodine. The conditions of the titration must be carefully controlled, since the relation between methylglyoxal present and iodine consumed is only empirical.

Within the limits of experimental error, the titrimetric and manometric procedures give identical glutathione values for extracts of animal and plant tissues. The titrimetric procedure is somewhat more time-consuming (six simultaneous determinations in 2.5 hours), but is quite simple from a manipulative viewpoint. Standard preparations of glyoxalase, which retain their activity for months, can be made easily from baker's yeast. Methylglyoxal solutions are made by distilling glyceraldehyde or dihydroxyacetone with dilute H_2SO_4, and may also be preserved unchanged for months. As developed at present, the titrimetric procedure permits the estimation of 0.01 to 0.10 mg. of glutathione in 0.5 cc. of tissue extract. With amounts above 0.025 mg., the limit of experimental error is about 5 per cent.; with smaller amounts, the probable error is larger. It is to be expected that with these highly specific methods available, the elucidation of the glutathione problem will be considerably facilitated.

Electrolytic Reduction and Determination of Oxidized Glutathione.—JANETTA SCHOONOVER DOHAN AND GLADYS E.

WOODWARD. (*J. Biol. Chem.*, **129**: 393, 1939. See also G. E. Woodward and E. G. Fry, *J. Biol. Chem.*, **97**: 465, 1932; G. E. Woodward, *J. Biol. Chem.*, **109**: 1, 1935; E. F. Schroeder and G. E. Woodward, *J. Biol. Chem.*, **129**: 283, 1939). For years, the accurate estimation of oxidized glutathione has constituted a difficult problem. The solution to be described here is the contribution of an organization which has devoted considerable attention to the study of glutathione and to the development of trustworthy methods for its determination. This paper is one of four published at various times from this laboratory on methods of determination of glutathione in its reduced ($HOOC.CH(NH_2).CH_2.CH_2.CONH.CH(CH_2SH).CONH.CH_2COOH$) and oxidized forms. Oxidized glutathione is formed from two molecules of reduced glutathione by loss of hydrogen from the –SH groups and formation of a linkage between the two S atoms.

Glutathione, in its reduced form, is a compound of such widespread occurrence in the plant and animal worlds that many investigators have felt that it must take part in one or more essential physiological reactions. From studies inspired by this belief it is now known that reduced glutathione can activate enzymes of such differing functions as the hydrolysis of proteins and the conversion of methylglyoxal ($CH_3.CO.CHO$) to lactic acid ($CH_3.CHOH.COOH$).

The need of a specific method for the determination of reduced glutathione was met by a publication from this laboratory describing a procedure in which use was made of the fact that reduced glutathione was the only known substance which activates glyoxalase (the enzyme which converts methylglyoxal to lactic acid). Up to that time, reduced glutathione determinations had been based, for the most part, upon the ability of reduced glutathione to take up iodine in acid solution, although it soon was recognized that this ability was shared by other biochemical compounds, as, for example, vitamin C and cysteine. The values obtained by iodine titrations, therefore, are frequently much too high.

On the other hand, very little is known about the occurrence and even less about the reactions of oxidized glutathione. The ease with which it is formed from the reduced compound *in vitro* and with which the reverse reaction

occurs in the presence of liver implies that such oxidations may take place in the organism. On this basis it has been believed that the ratio of the oxidized to the reduced form reflects, or perhaps even regulates, the oxidation-reduction balance of the tissues.

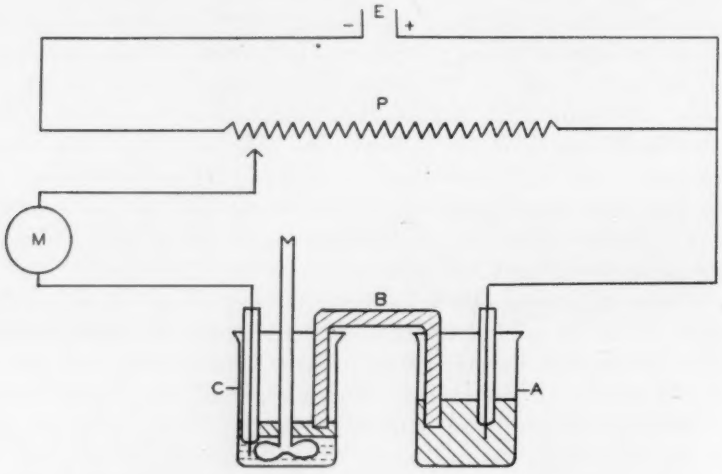
The scarcity of studies on oxidized glutathione probably is due, at least in part, to the lack of a reliable method for its estimation. Since no method has been found for measuring oxidized glutathione directly, it is usual to estimate it in its –SH form after reduction. The accuracy of its determination therefore depends upon (1) the completeness of reduction, and (2) the specificity of the method used for estimation of reduced glutathione. No method heretofore available has given satisfaction in both respects. For instance, the most familiar reduction procedure, involving the interaction of zinc and acid, does not give complete reduction in the presence of protein-free filtrates of biological materials, and the zinc ions in the resultant solution are toxic to glyoxalase.

These difficulties now have been overcome. Electrolytic reduction in an acid medium with a mercury cathode has been found to change oxidized glutathione to reduced glutathione rapidly and completely; the resulting solution has been found to be suitable for glutathione estimation by the specific glyoxalase method as well as by the iodometric procedures.

Figure 1 shows a diagram of the reduction system. Direct current is supplied from any convenient source (*E*). The voltage is regulated by a slide-wire potentiometer (*P*) and the current passing through the electrolytic circuit is measured by a milliammeter (*M*). Connection is made with the cathode, a layer of mercury in the bottom of a small beaker or a test-tube (*C*), by means of a platinum wire electrode dipping below the surface of the mercury. Over the mercury is placed the solution to be reduced. The mercury or mercury-solution interface is stirred mechanically by means of a small glass paddle stirrer. A KCl-agar salt bridge (*B*) is used to make connection with the anode compartment (*A*) which contains 4 per cent. sulphosalicylic acid. Into the acid dips the anode itself, another platinum wire electrode. Connection with the potentiometer then completes the circuit.

For each reduction, the proper amount of mercury and the solution to be treated are placed in the cathode vessel; when the latter and the salt bridge are placed in position, the circuit becomes complete. Stirring is started and adjusted to the proper speed. The current is turned on, regulated by the potentiometer to produce the best current density, and allowed to flow through the circuit for 10 to 20 minutes.

FIG. 1.



Wiring diagram for reduction. (Courtesy, *Journal of Biological Chemistry.*)

After current and stirring have been stopped, the solution is pipetted off and analysis for reduced glutathione is made directly on the solution without further treatment. In actual practice, all of the apparatus remains assembled except the cathode vessel and salt bridge, which are closed each time according to the volume of solution to be reduced. The cathode vessel should be selected so that the depth of solution is about 0.5 to 0.7 cm. The salt bridge should have as wide a bore as possible, in order to avoid heating, but with small cathode vessels a narrower one must be used to give clearance to the stirrer.

It was found that there was considerable latitude in the concentration of oxidized glutathione that could be reduced, in the acidity of the solution, in the current density (current

per unit area of the mercury cathode surface), and in the time of reduction.

By the combined use of electrolytic reduction and the glyoxalase method, no oxidized glutathione was found in rat liver, spleen, heart, muscle, or blood, or in rabbit blood. These negative results are in disagreement with the former belief that it would be found in most tissues. It is possible, of course, that oxidized glutathione may be found in some other normal tissue or in pathological tissue.

The method should find extensive application in studies of the behavior of oxidized glutathione or in studies of reduced glutathione where it is important to ascertain the extent of any loss due to oxidation. It has enabled us, for example, to explain the poor recovery of oxidized glutathione which had been added to blood when zinc and acid were used for reduction. It was formerly assumed that oxidized glutathione was bound to the protein and did not pass into the protein-free filtrate. Electrolytic reduction, however, shows that all of the added oxidized glutathione was in the filtrate. There appears to be some condition of the filtrate which leads to the inhibition of the zinc reduction. The observation that reduced glutathione added to plasma, serum, or hemolyzed blood disappeared and could not be recovered even after zinc reduction may be explained on the same basis. Reduced glutathione added to plasma or serum rapidly becomes oxidized, and in the oxidized form would not be recovered by this method. It is entirely recovered after electrolytic reduction.

BOOK REVIEW.

THE FUNDAMENTALS OF ELECTRO-MAGNETISM, by E. Geoffrey Cullwick. 352 pages, illustrations, 14 × 23 cms. New York, The Macmillan Company, 1939. Price $4.50.

One of the results of the New Physics revelations is the recognition by forward looking teachers of scientific subjects of the need not only of presenting their subject matter in this light but in so doing, to present it in a more definite and digestible manner. To impart truth as closely as is known in simplicity certainly is a mark worthy to aim at. This book is an attempt in that direction. It is a change of viewpoint toward the subjects of electricity and magnetism and methods of presenting them. In it, the physical viewpoint is adopted as contrasted with the usual basis on the ideas of Faraday and Maxwell, and being free from unnecessary artificialities, it is that of "action at a distance, retarded in time." In other words the mutual actions of electric charges are looked upon as the basis of the phenomena. It forsakes the attempt to explain the mechanism of these effects by means of the properties of an electro-magnetic field whose physical existence cannot be proved. Electric and magnetic fields are retained, not as physical realities, but as extremely useful mathematical vector concepts, and to these are added, in exactly the same category, the vector potential.

On this foundation, the book is divided into the subjects, in order, the electrostatic field and the electric current, the magnetic field and electromagnetic induction, the magnetic field of the electric current, ferromagnetism, and electromagnetic waves. Observable facts are reduced to theory and guards are placed against the endowing of hypothetical concepts, of which there are many made use of, with physical reality.

The method set forth is new, it is untried, and while it seems to have much in its favor, there is considerable opportunity for criticism. The high quality of thought behind it and the unquestionable exhaustive work to make this thought a reality certainly warrants serious consideration of the book. Teachers of experience will easily find much of interest and practicing engineers who have familiarity with the subject may be helped.

R. H. OPPERMANN.

ELEMENTS OF THE TOPOLOGY OF PLANE SETS OF POINTS, by M. H. A. Newman. 221 pages, illustrations, 15 × 23 cms. New York, The Macmillan Company, 1939. Price $3.50.

This book has the double purpose of providing an introduction to the methods of Topology, and of making accessible to analysts the simple modern technique for proving the theorems on sets of points required in the theory of functions of a complex variable. The two-dimensional theory is particularly well suited to form a first introduction to Topology, since it shows clearly the necessity for an algebraical treatment of complexes without involving any appeal to the theory of groups.

The principal subject-matter is the "combinatorial" properties of plane sets of points, that is, properties that depend essentially on the fact that a plane can be cut up into polygonal pieces. These properties include Jordan's Theorem on the division of the plane by a simple closed curve, and its generalizations, the properties of simply-connected domains, accessibility of the frontier, and connectivities of open sets and of simple closed sets. The necessary general theorems on sets of points (closed and open sets, connected sets, etc.) are collected in a preliminary section; they are established for sets in general metric spaces, since there is no greater difficulty in doing so.

R. H. OPPERMANN.

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